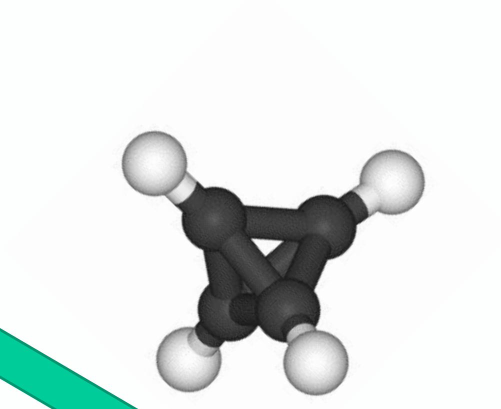
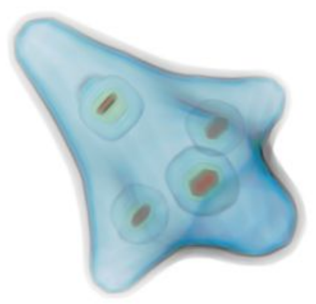
Introduction to topology-based analysis

Attila Gyulassy
SCI Institute, University of Utah

Why do we do topological analysis?

Data



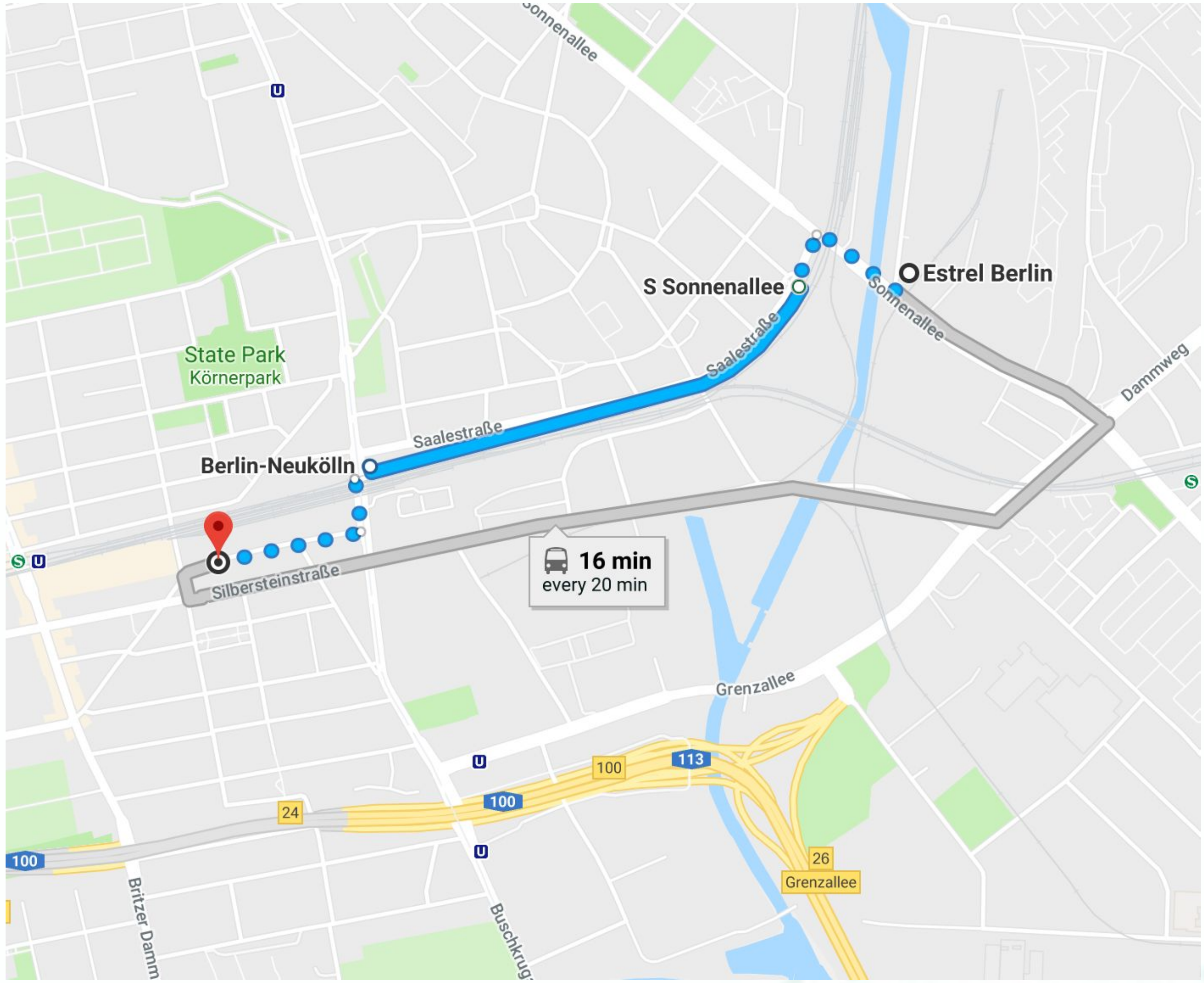


C_4H_4

Tetrahedran
e

Language

We need the right abstraction for the task



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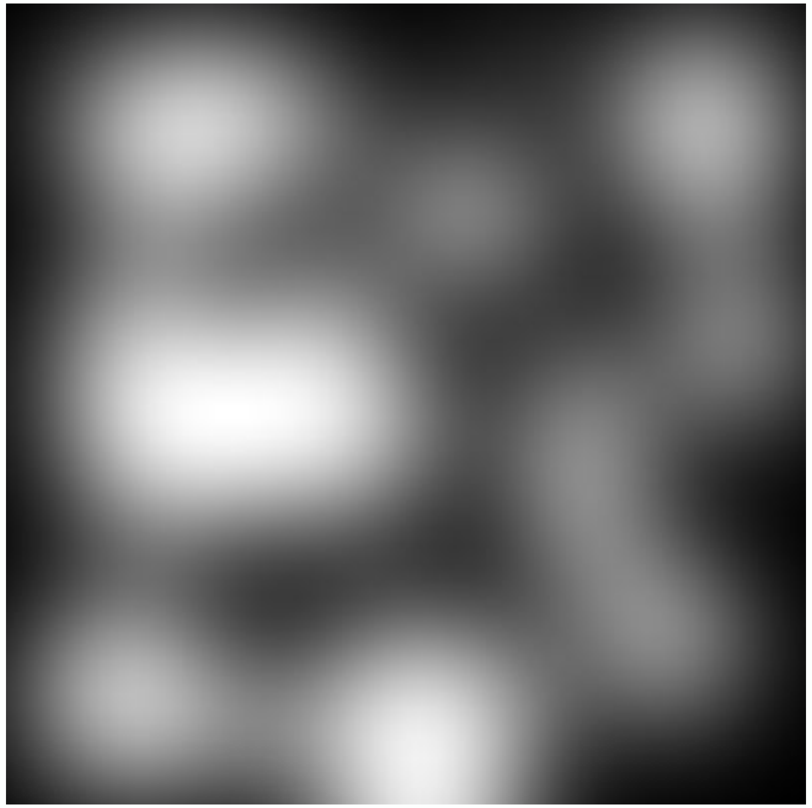
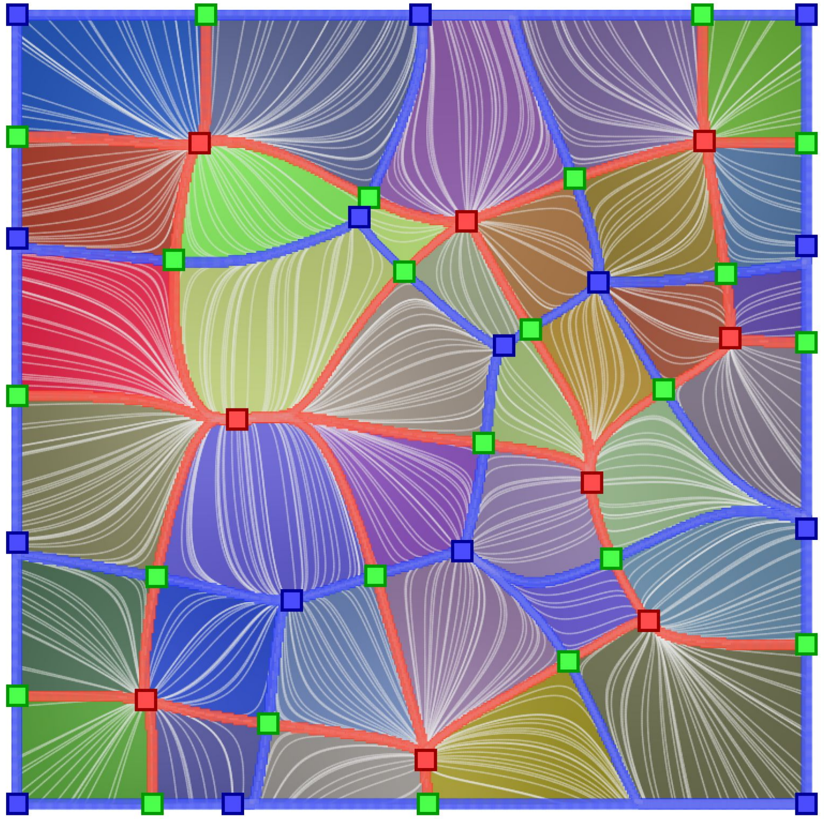
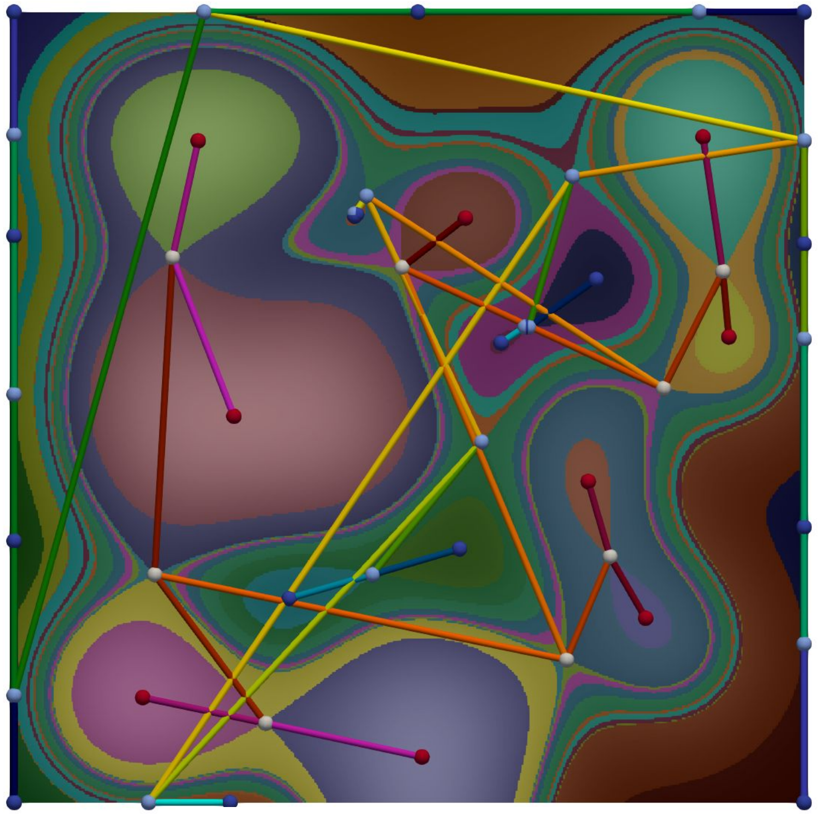
How do we make a map for data?



Topologial approaches provide a computable map

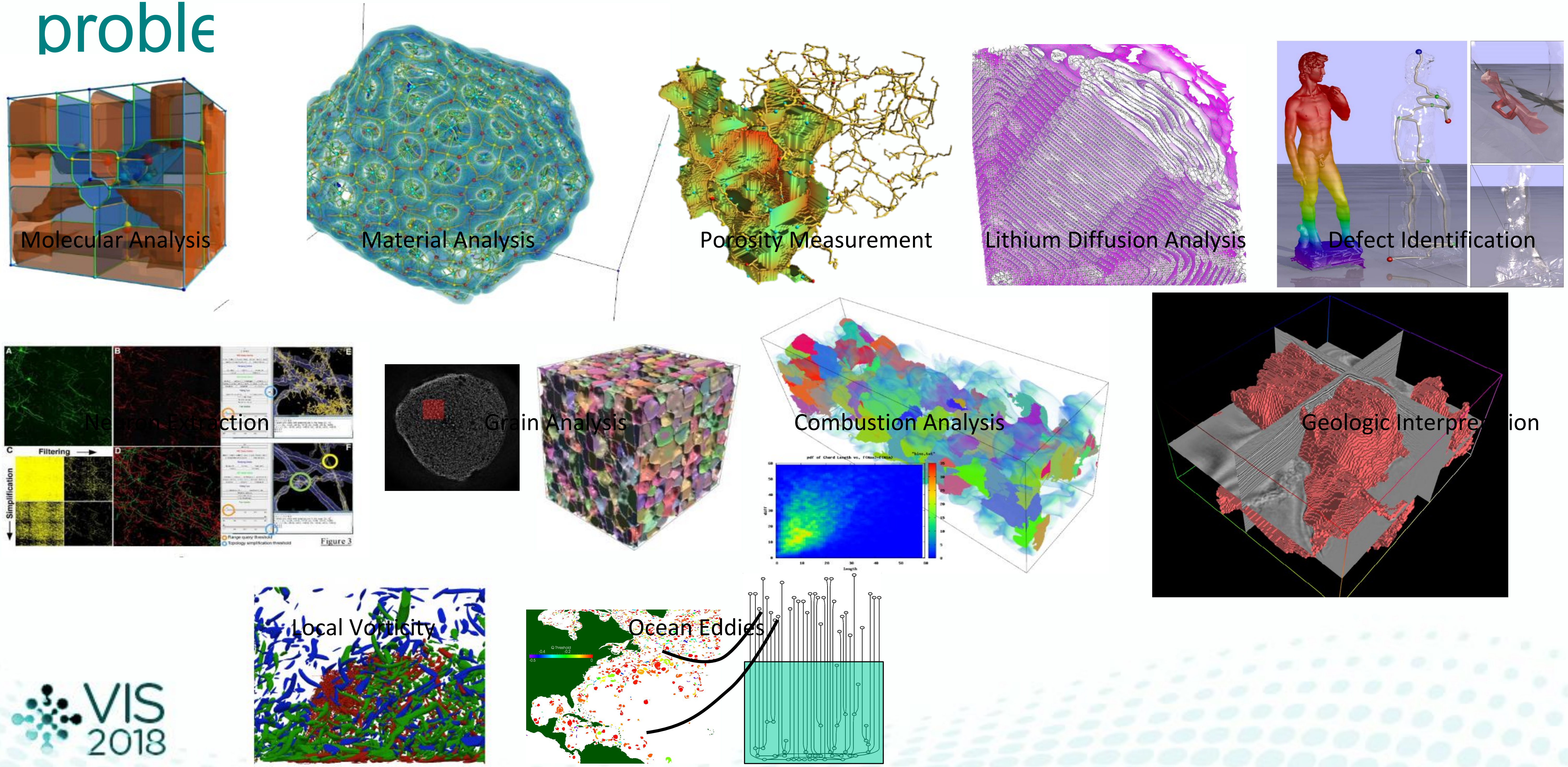


Morse-Smale Complex
Gradient based features



Contour tree
Contour based features

Topological abstractions can solve varied proble



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Analysis workflow

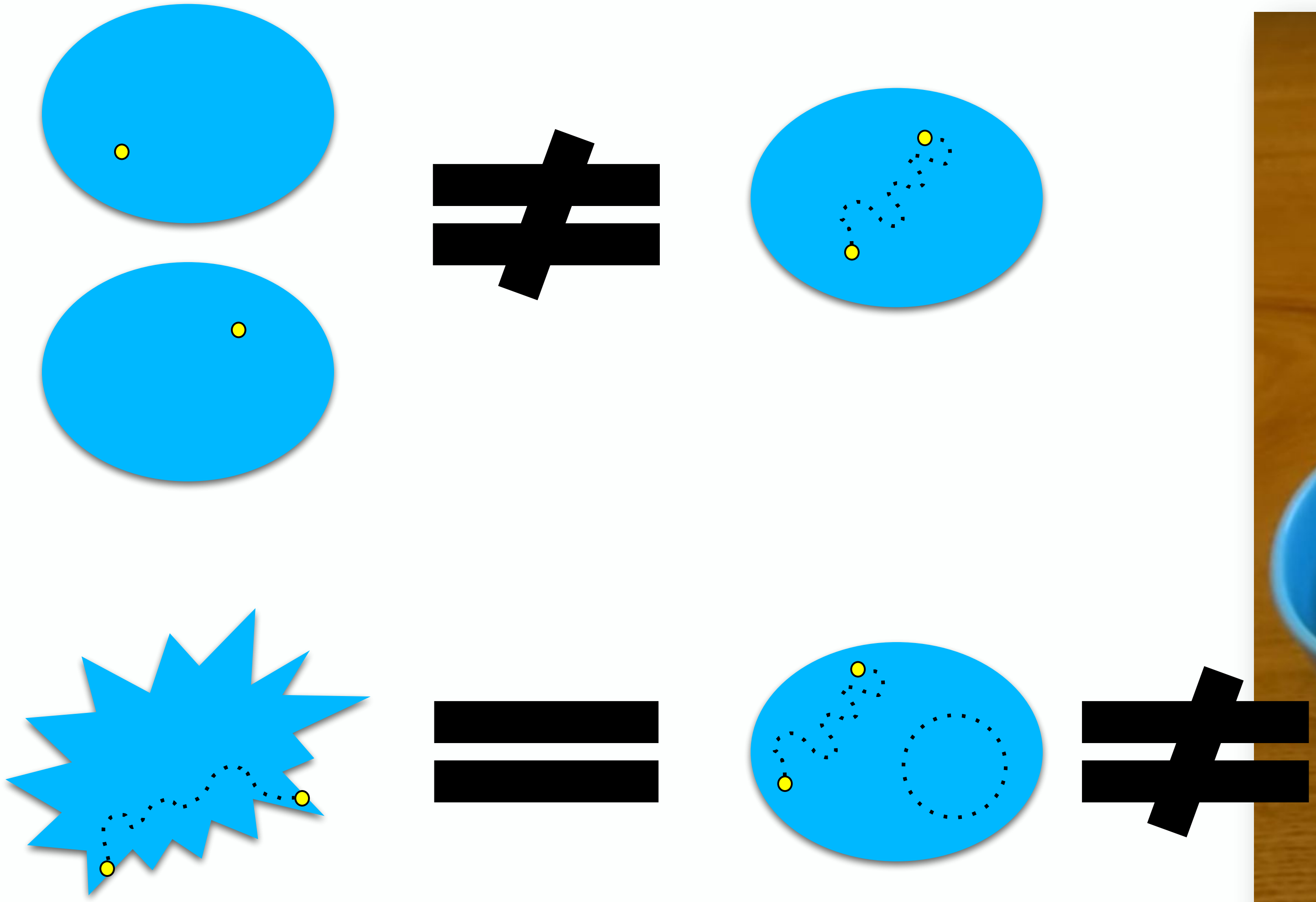
Generate data – simulation, imaging

Pick the right abstraction – compute mt, ct, ms complex, pd

Define the features –

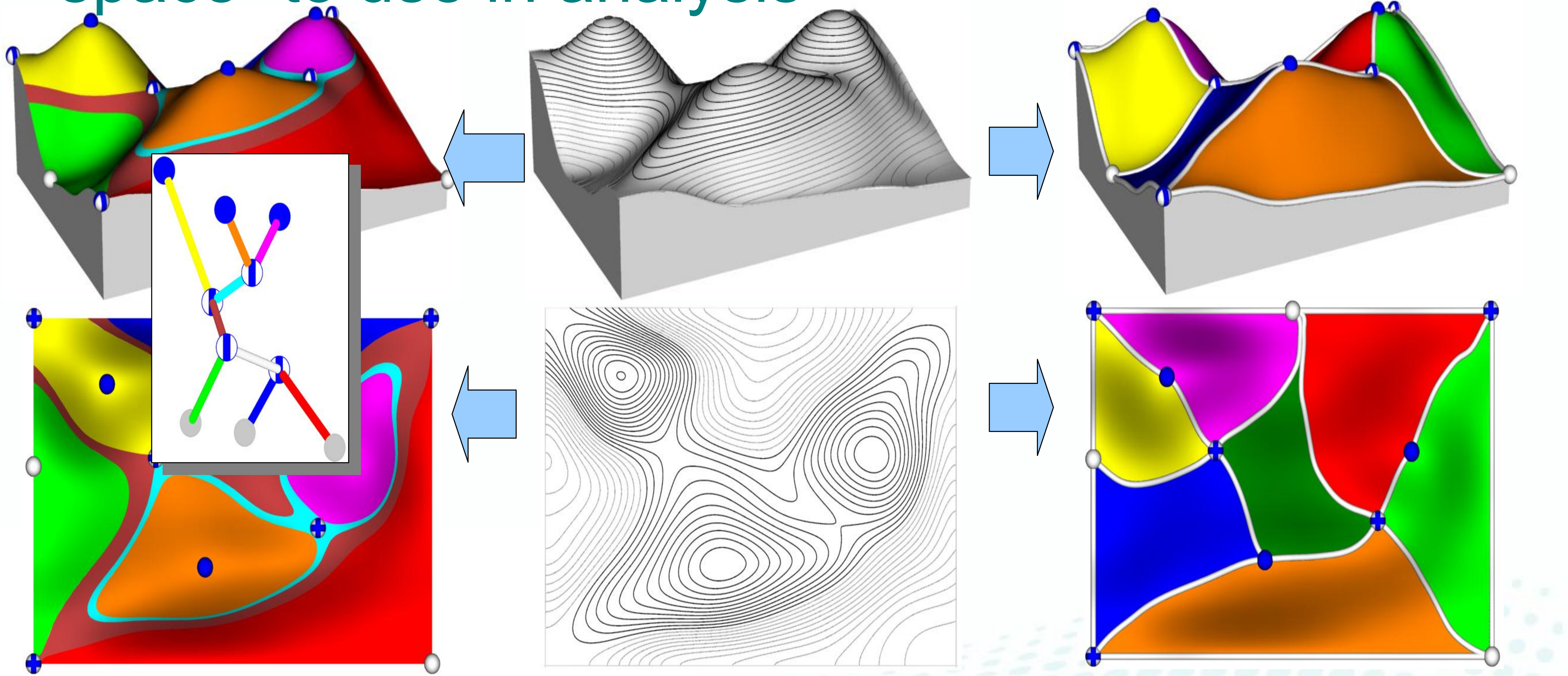
Evaluate (vis) features – repeat

“Topological” analysis is a broad term relating to connectedness, arrangement, and cycles



Art by Henry Segerman and Keenan Crane

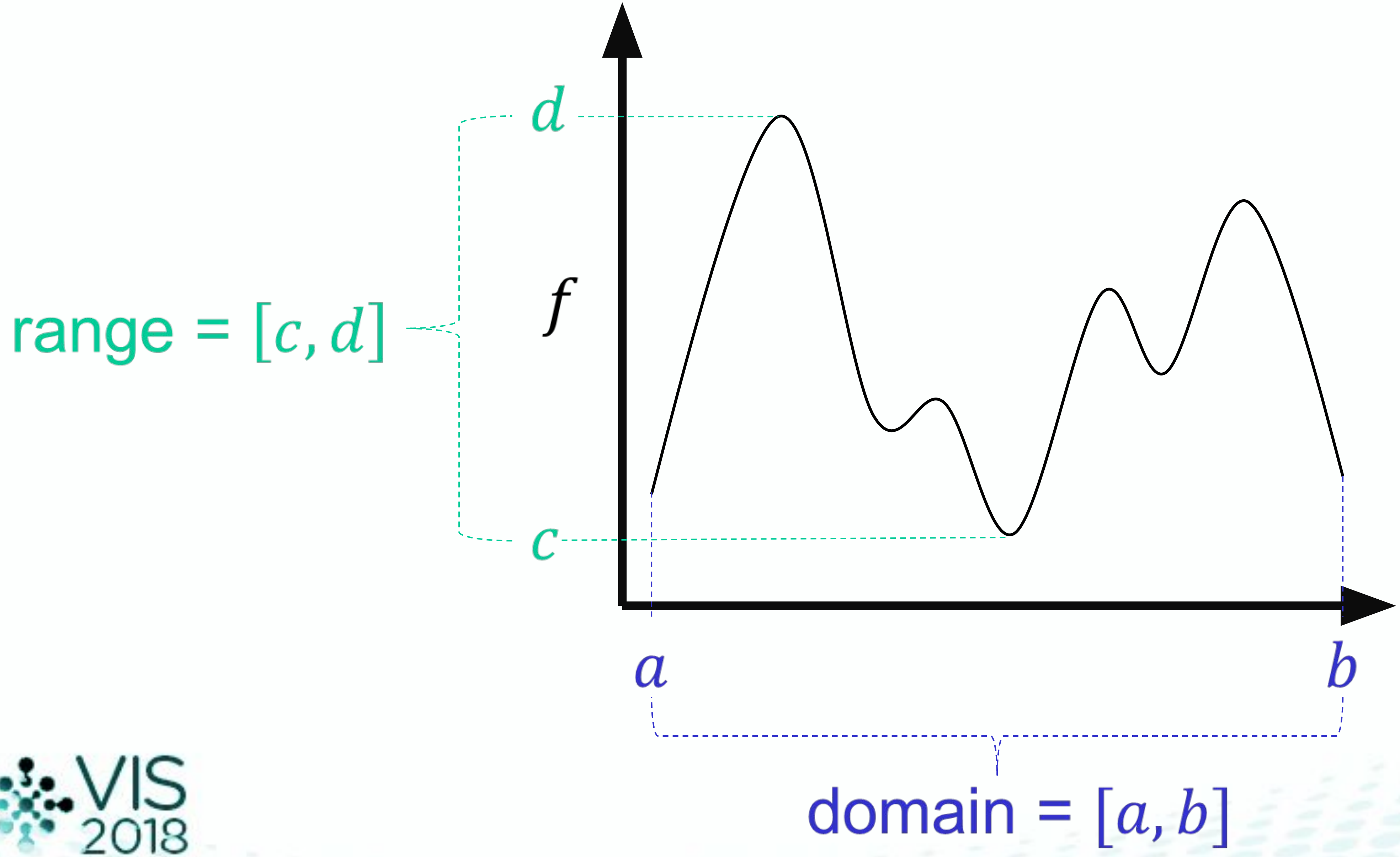
“Topological” approaches provide a “feature space” to use in analysis



Contour Trees/Reeb Graphs/Merge Trees

Morse/Morse-Smale Complexes

The “topology” of a scalar function relates to topological changes during a *filtration*



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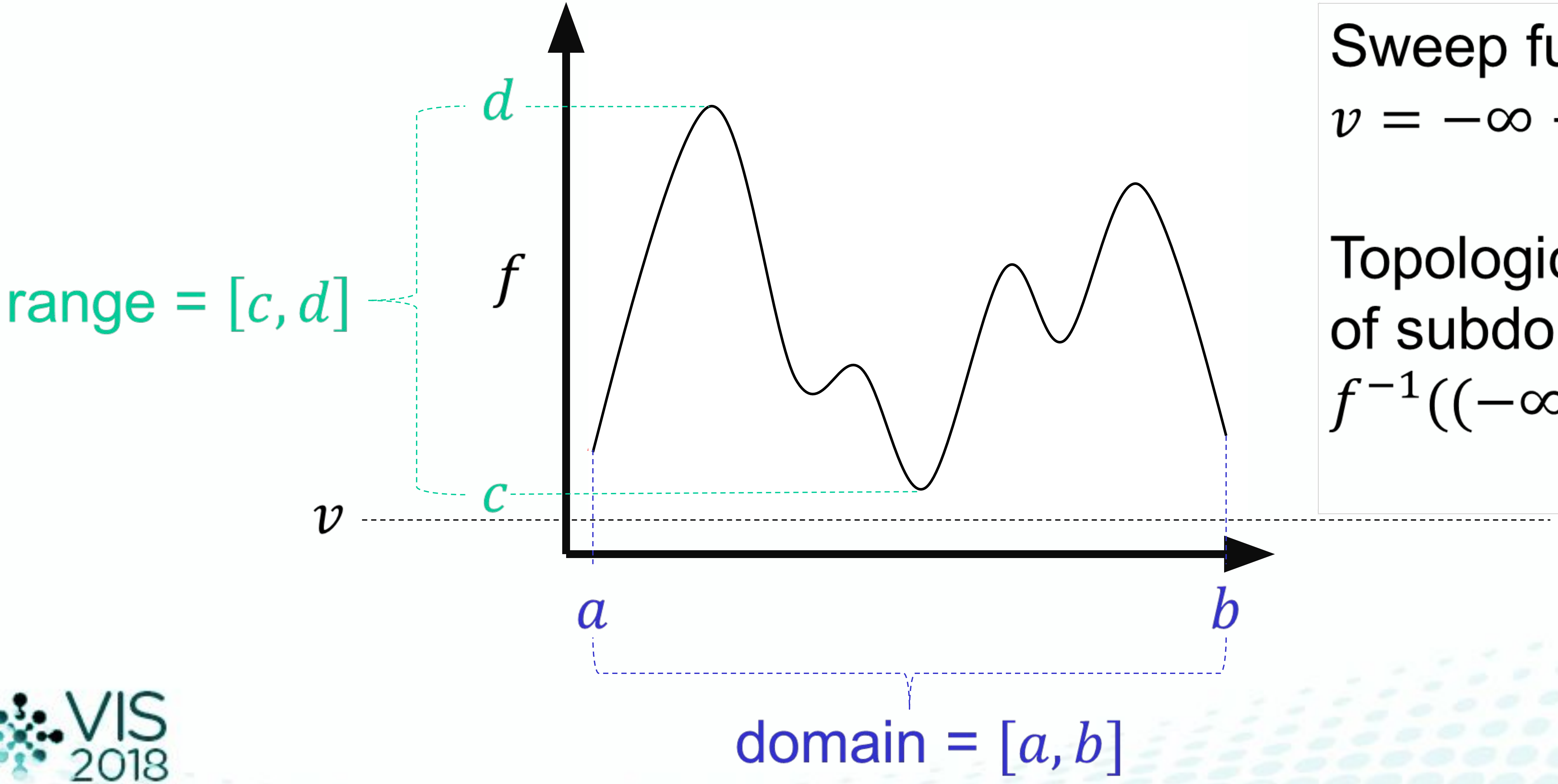

Sweep function from $v = -\infty \to \infty$

Topological changes of subdomain $f^{-1}((-\infty, v])$

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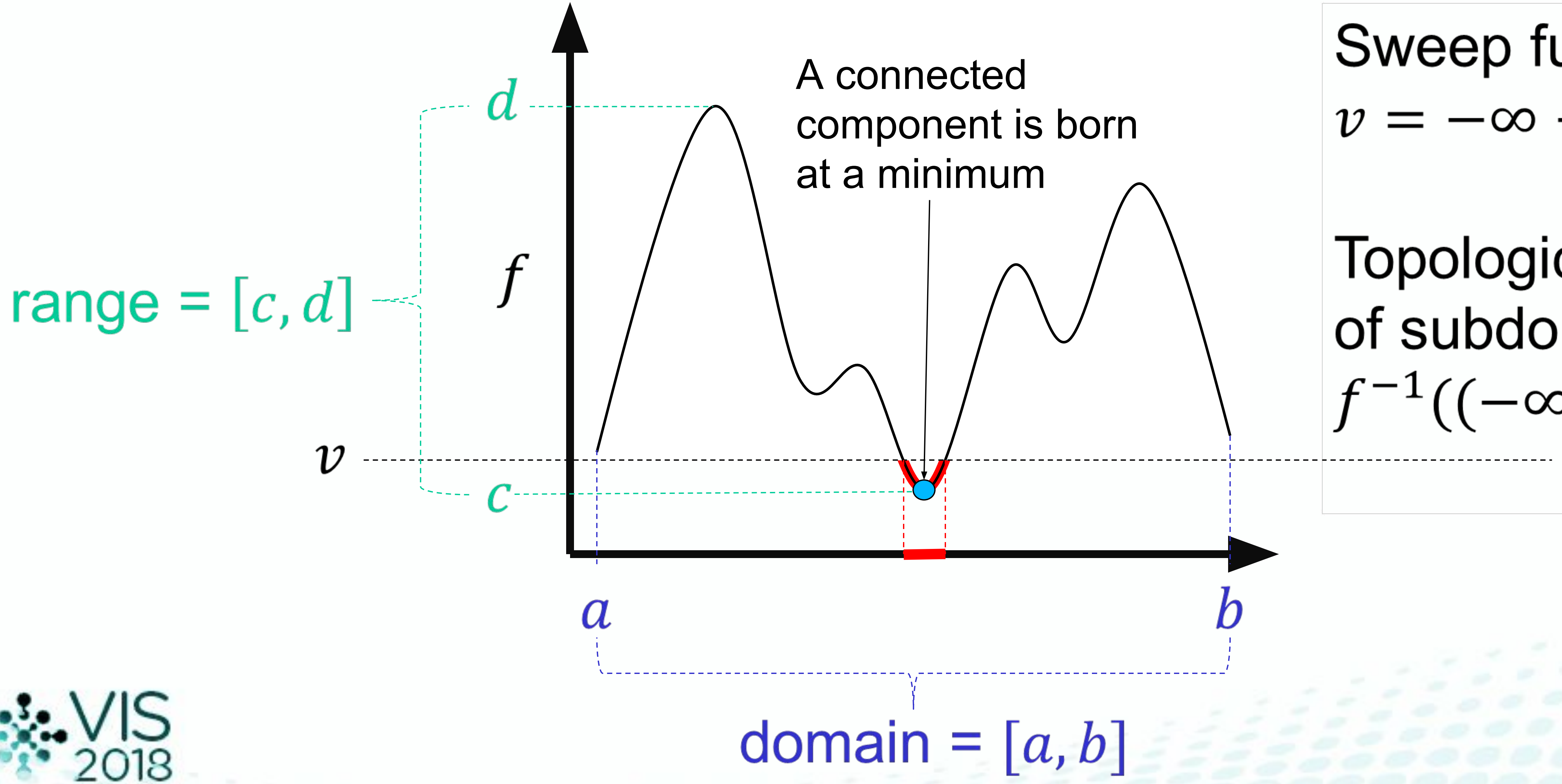

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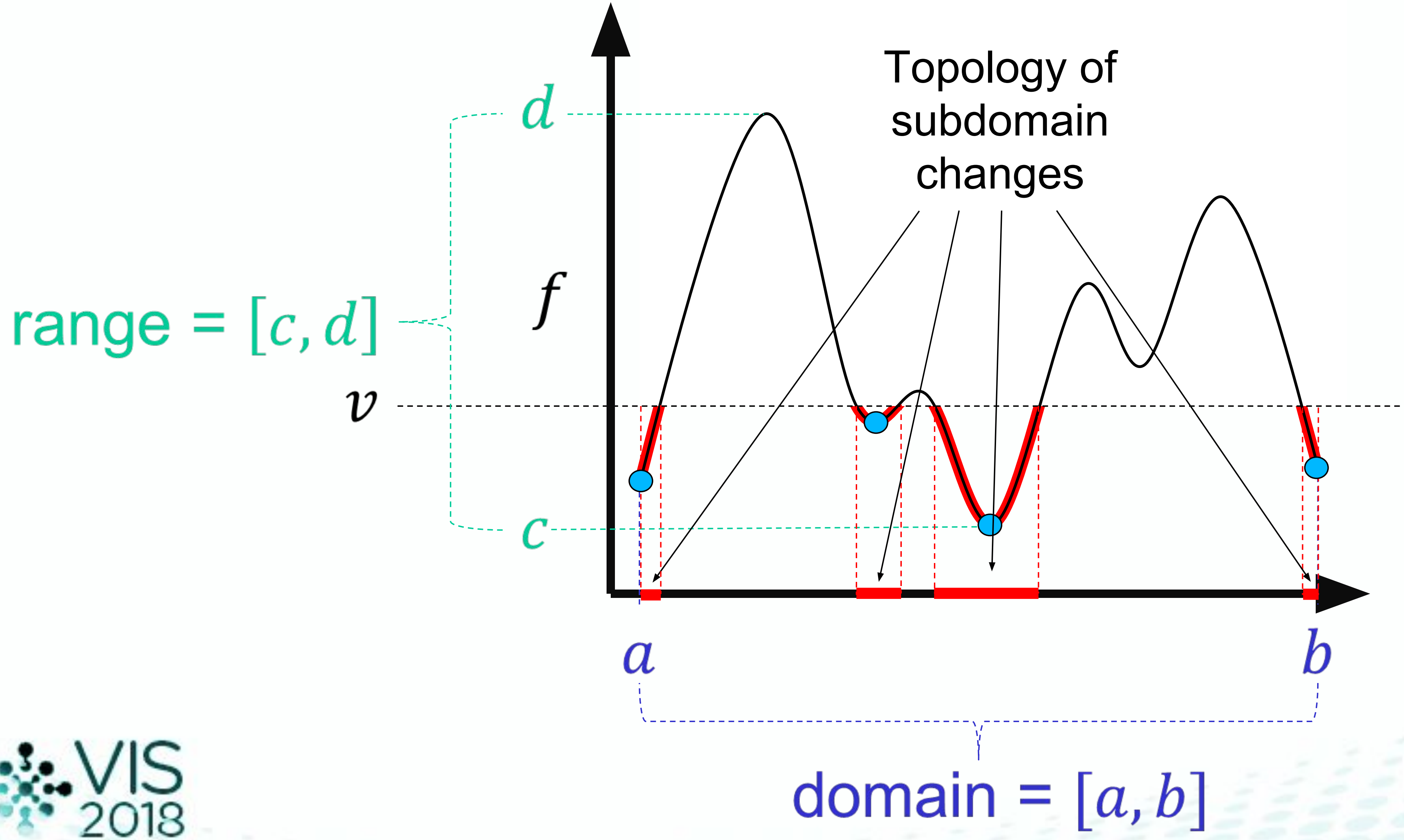

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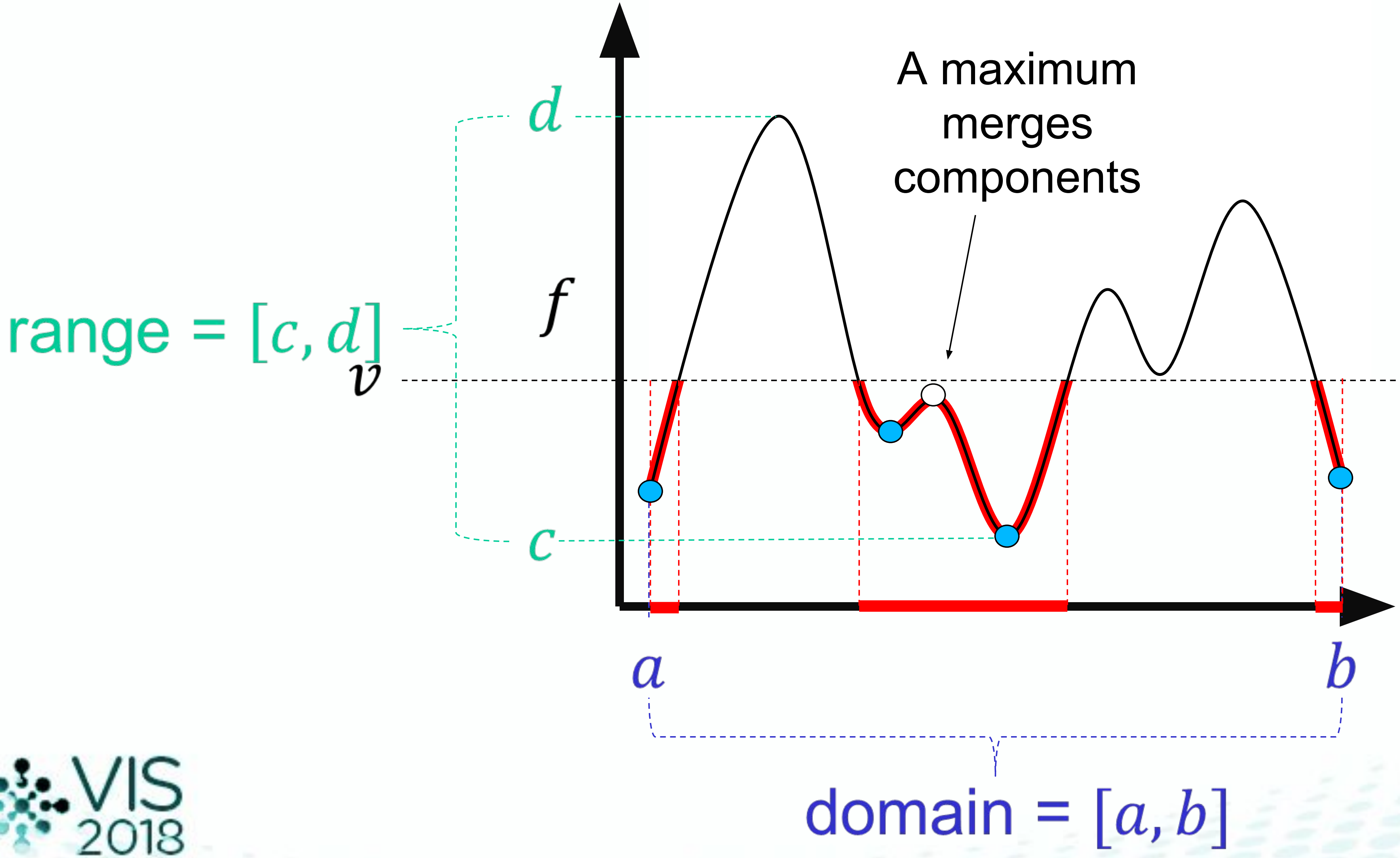

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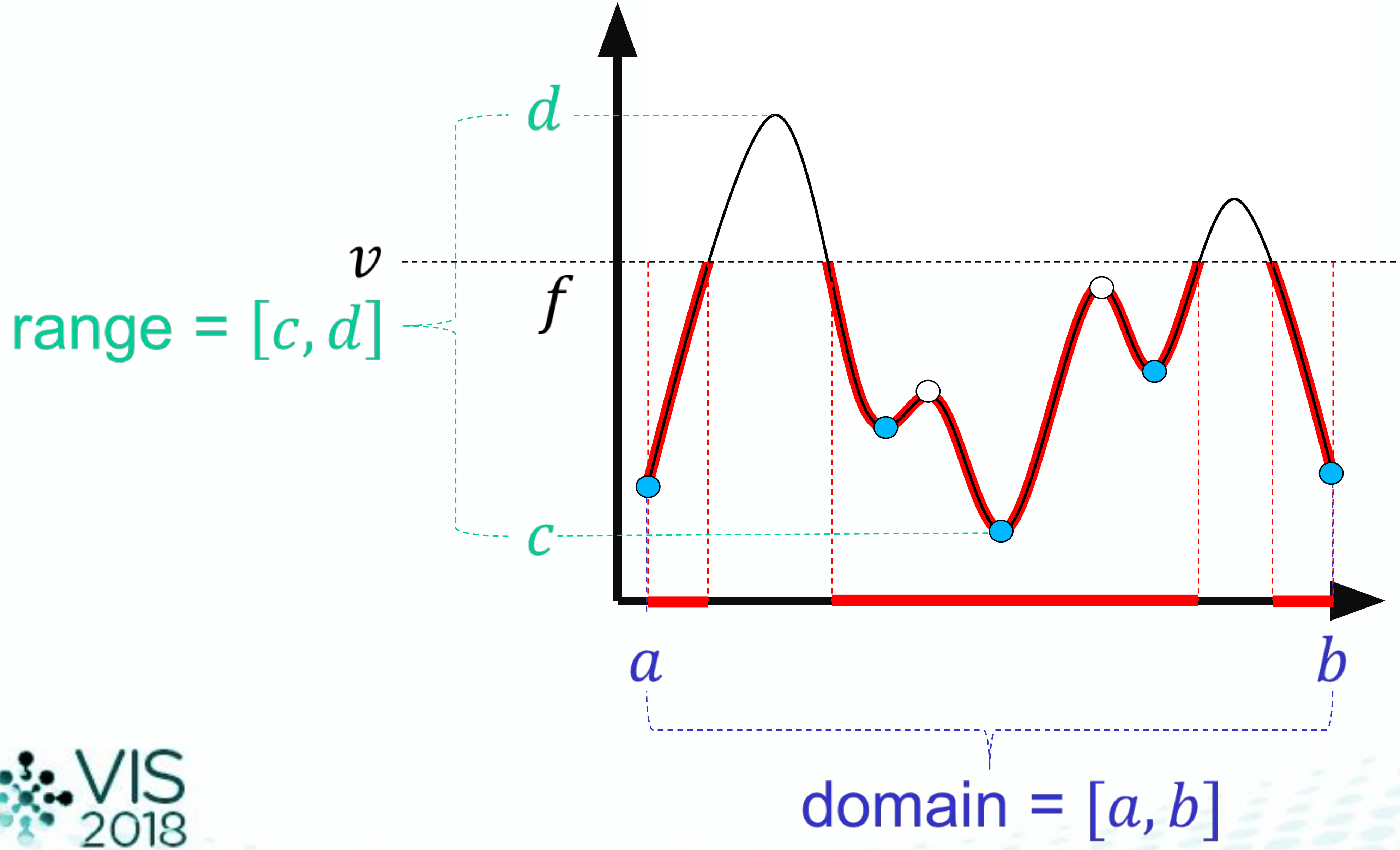

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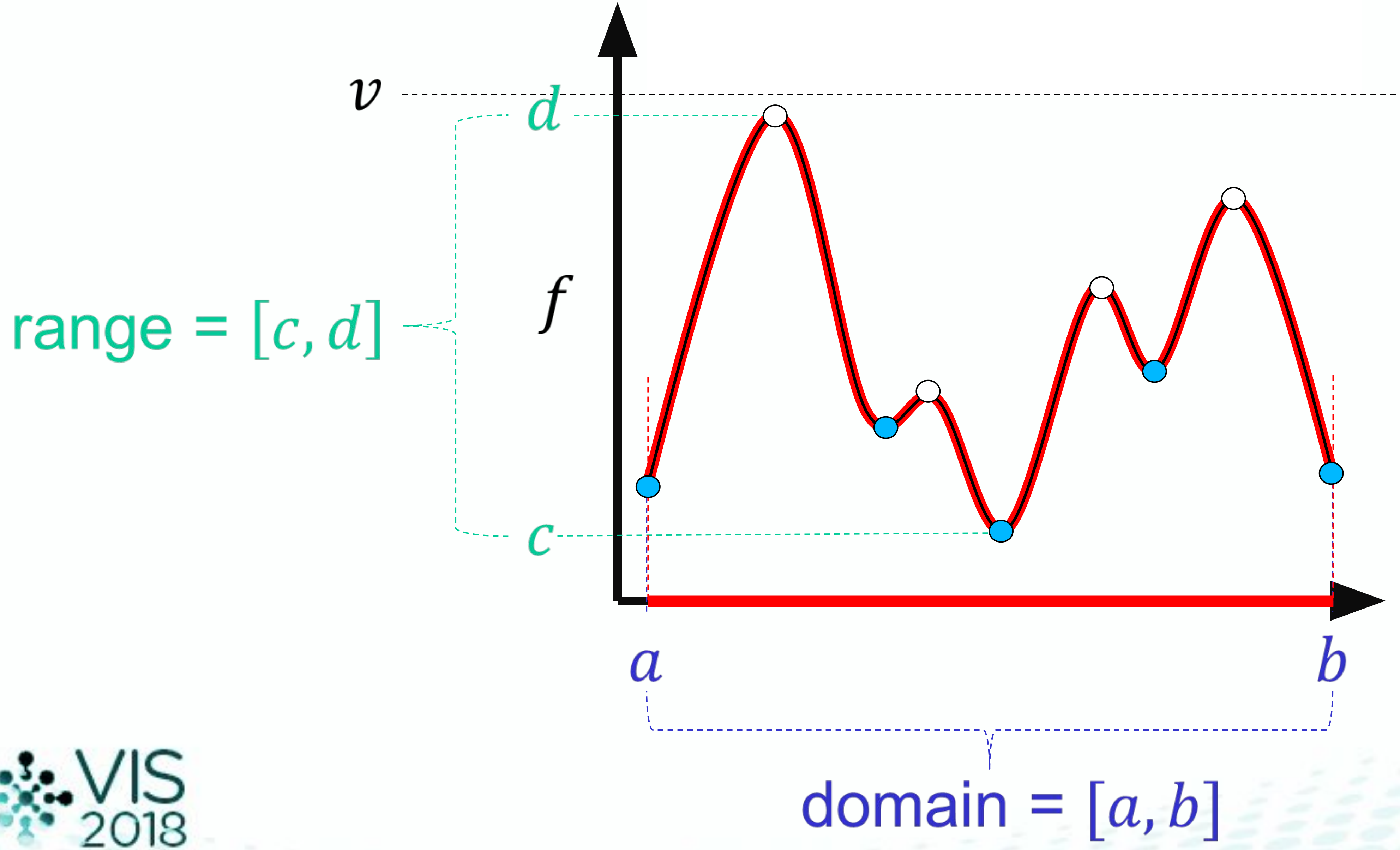

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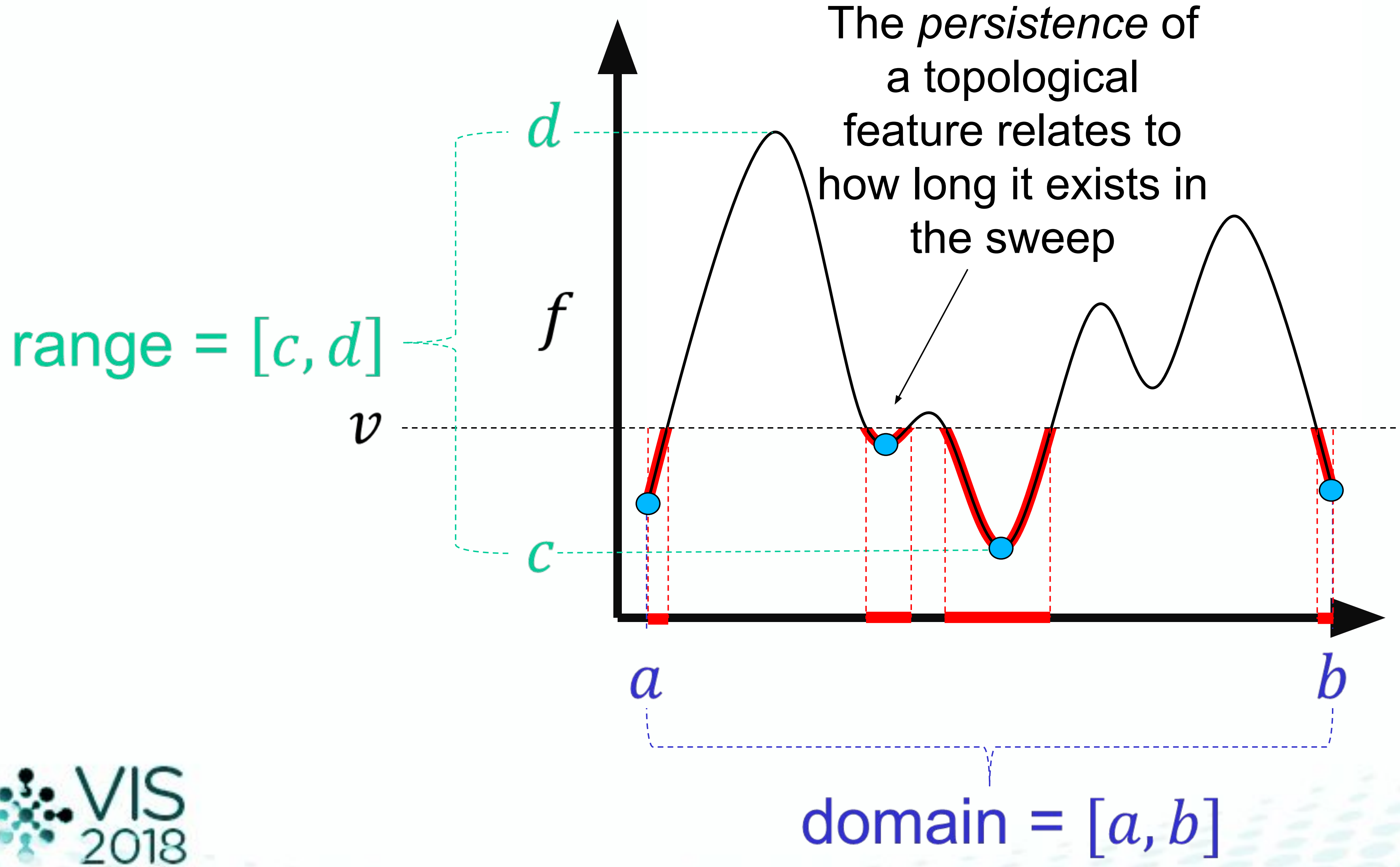

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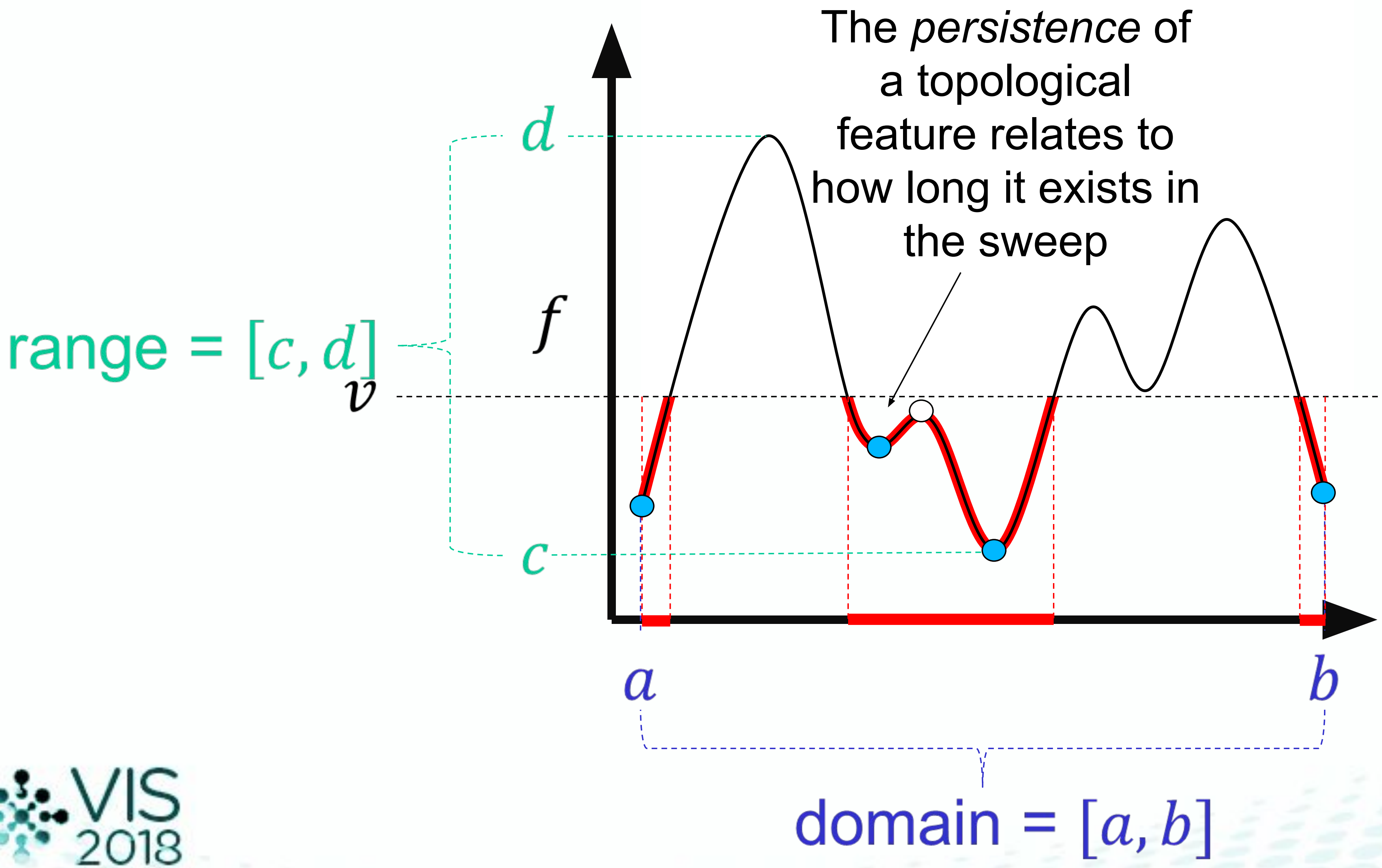

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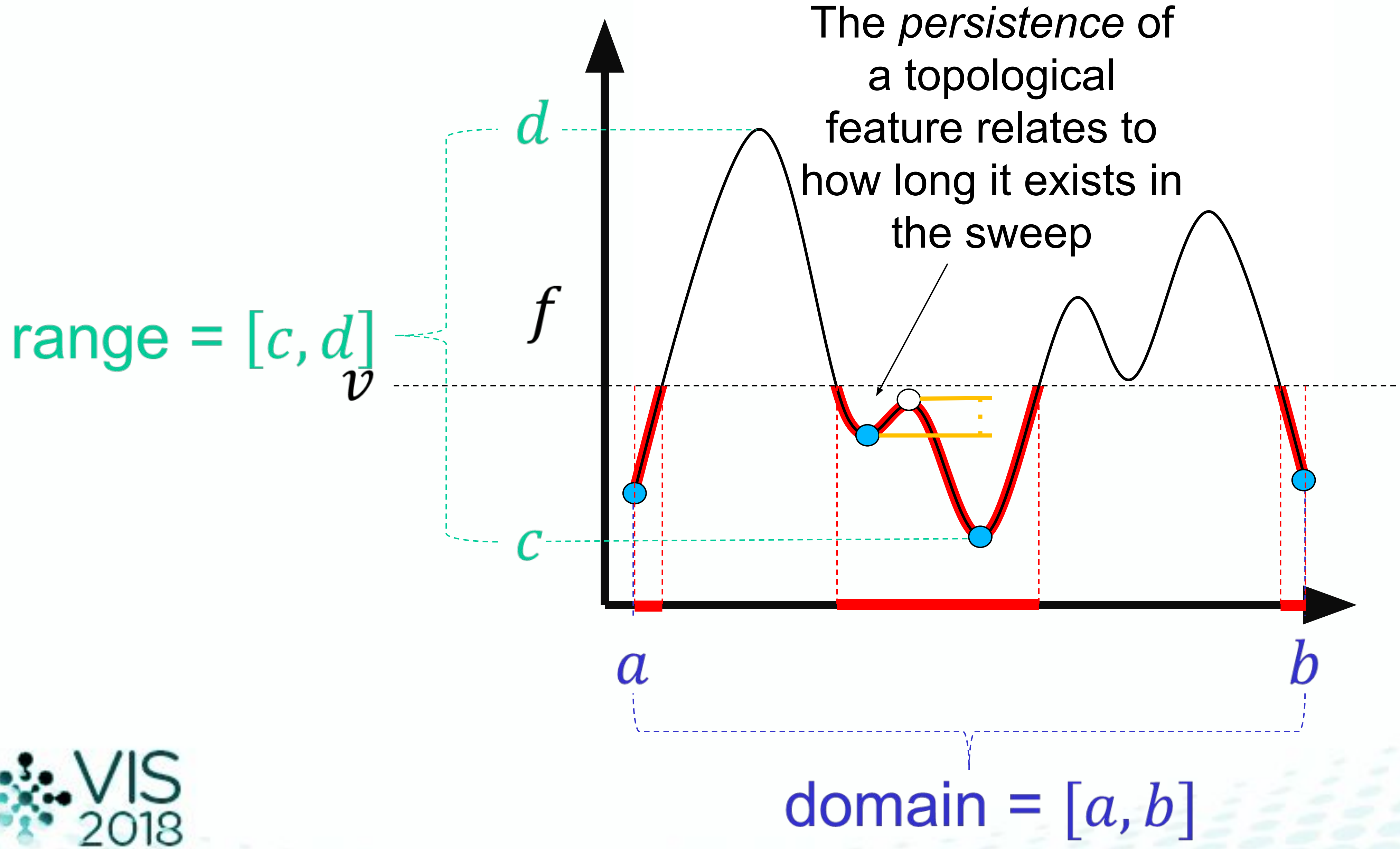

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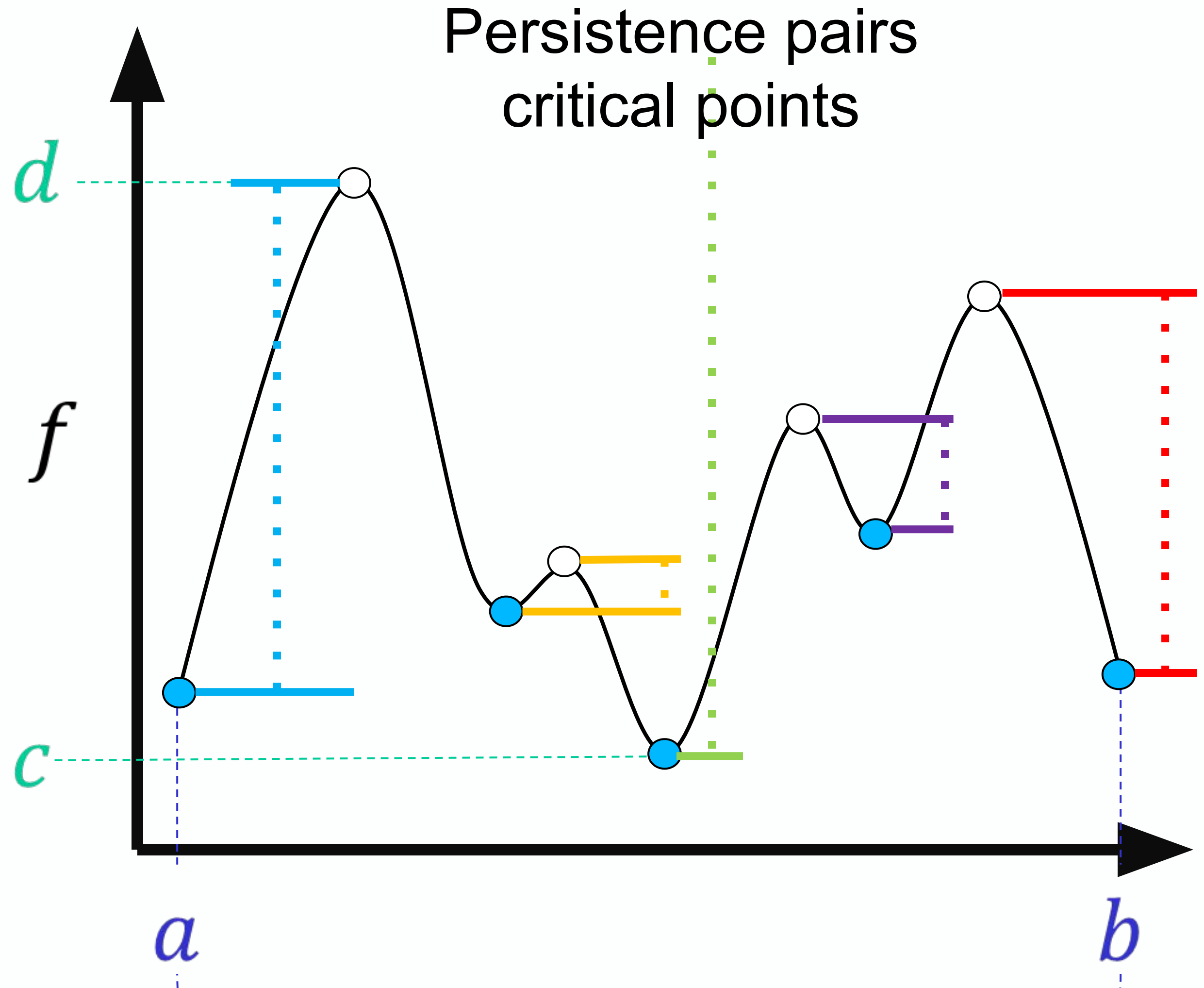

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domain = $[a, b]$

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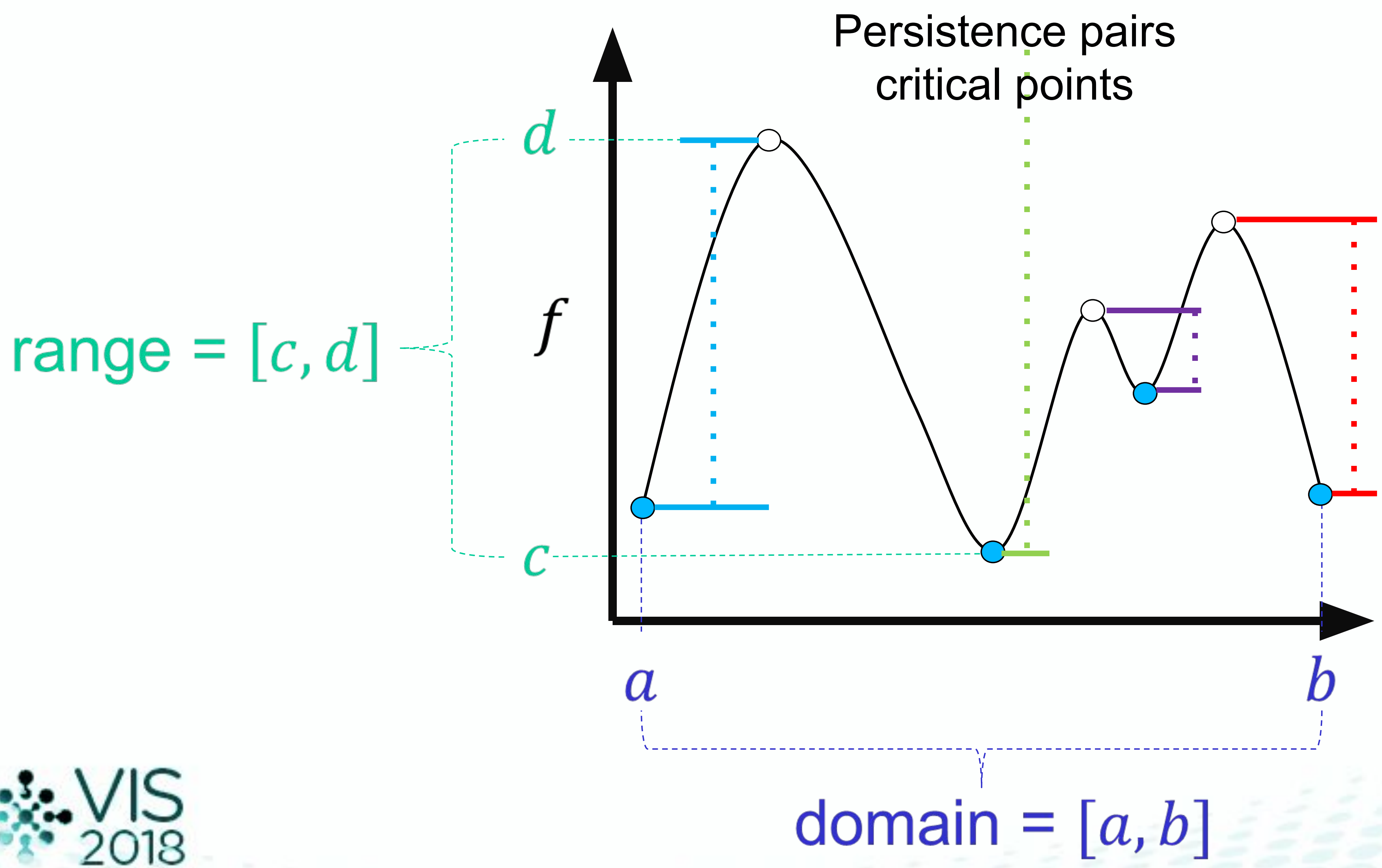

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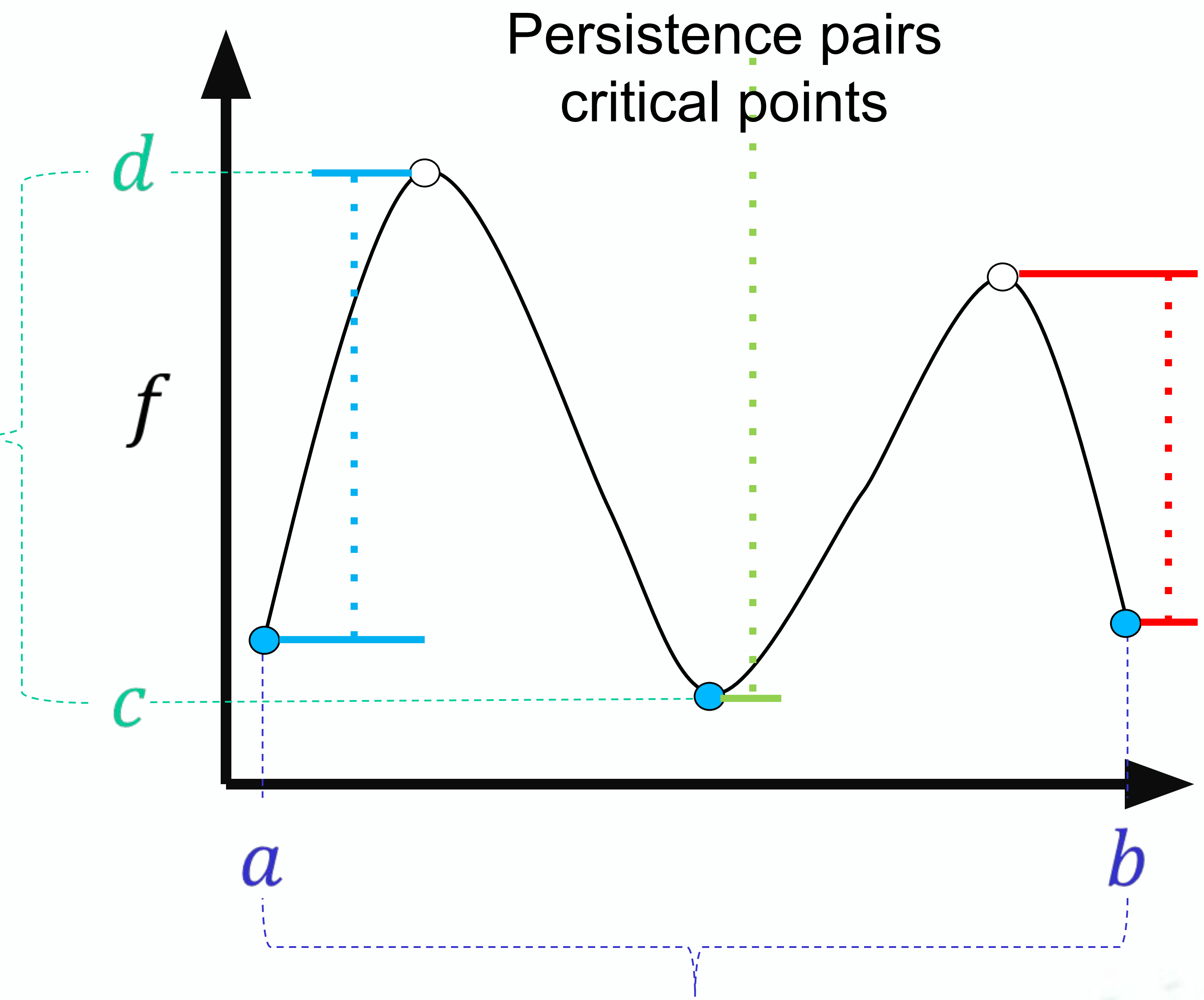

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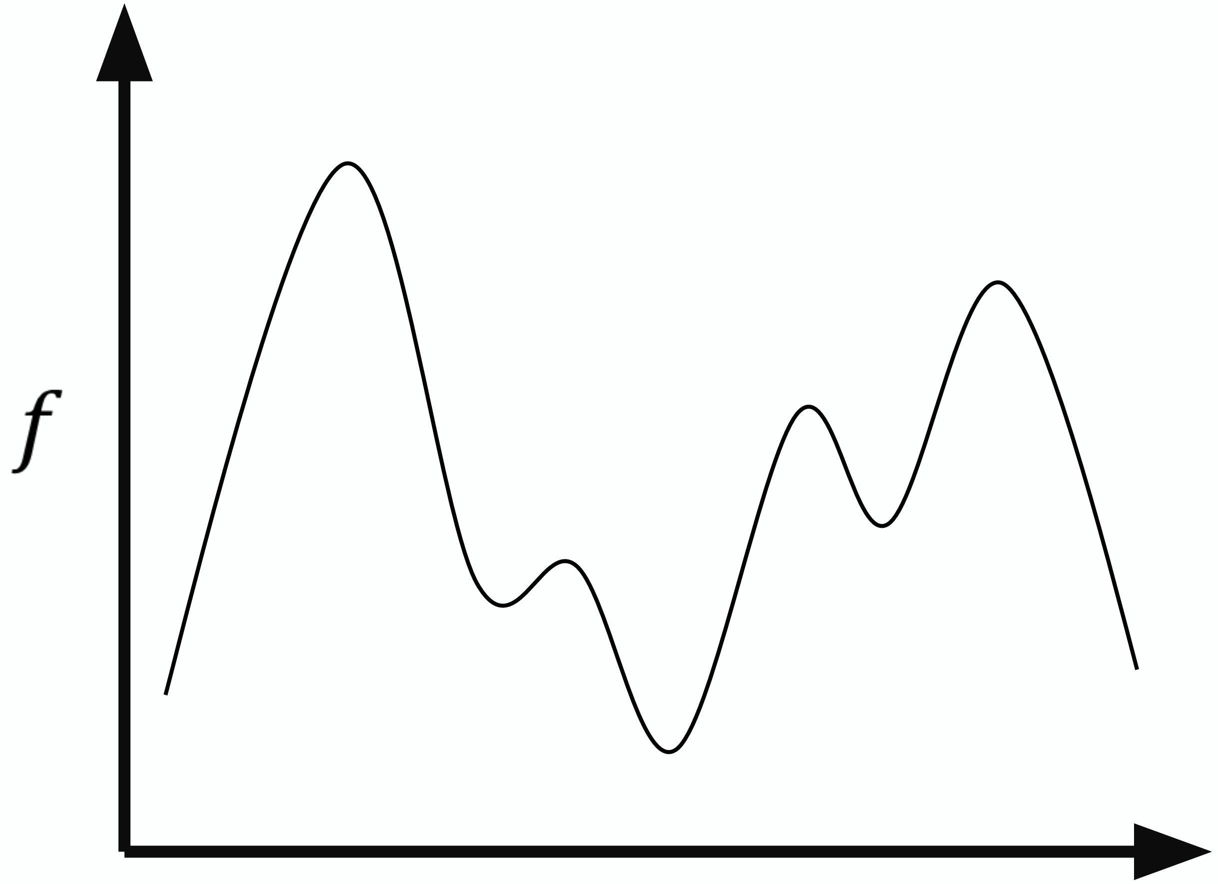
Features of a 1-dimensional function



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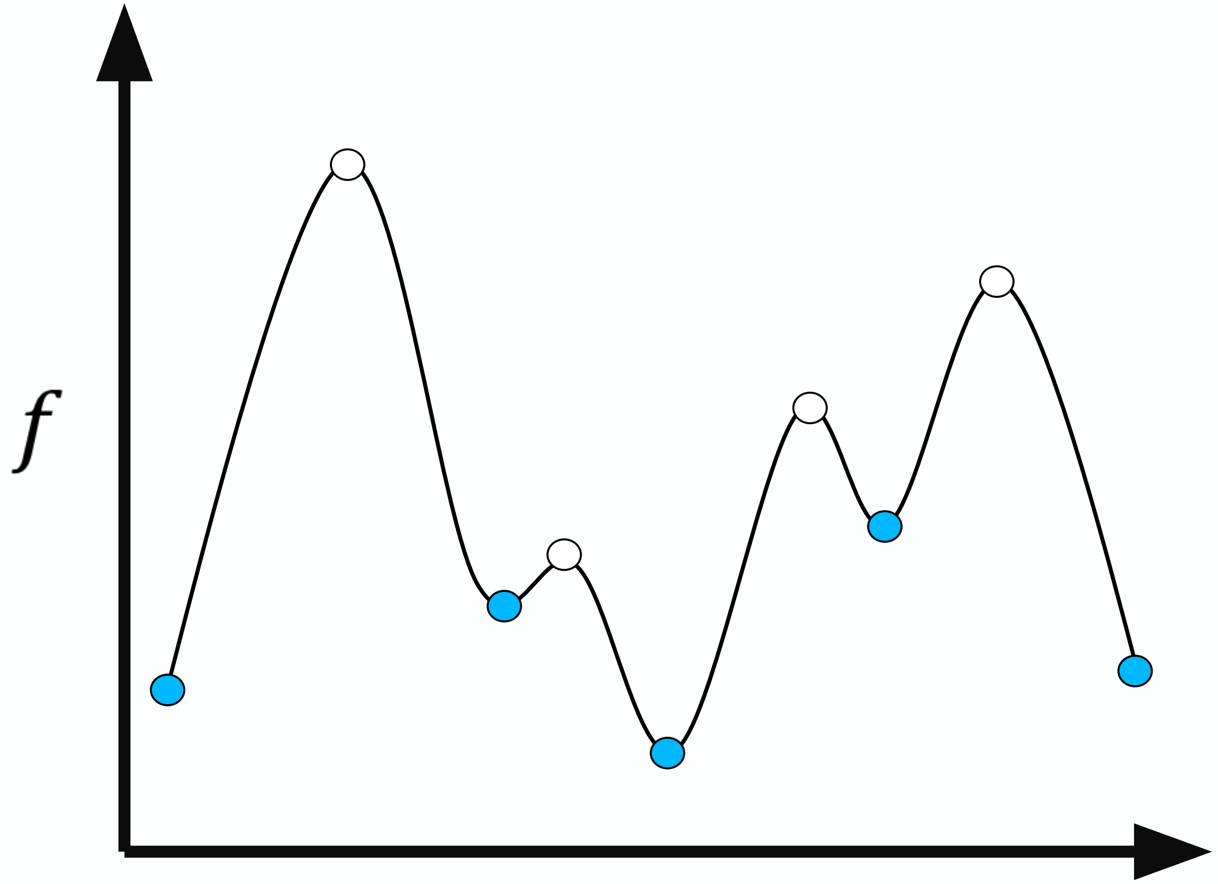
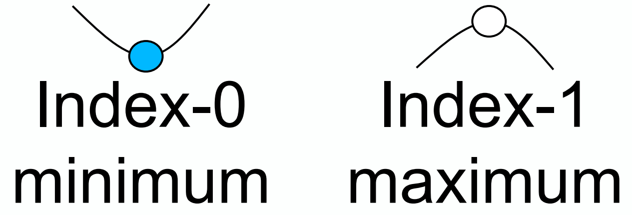


- Critical points where $\nabla f = 0$



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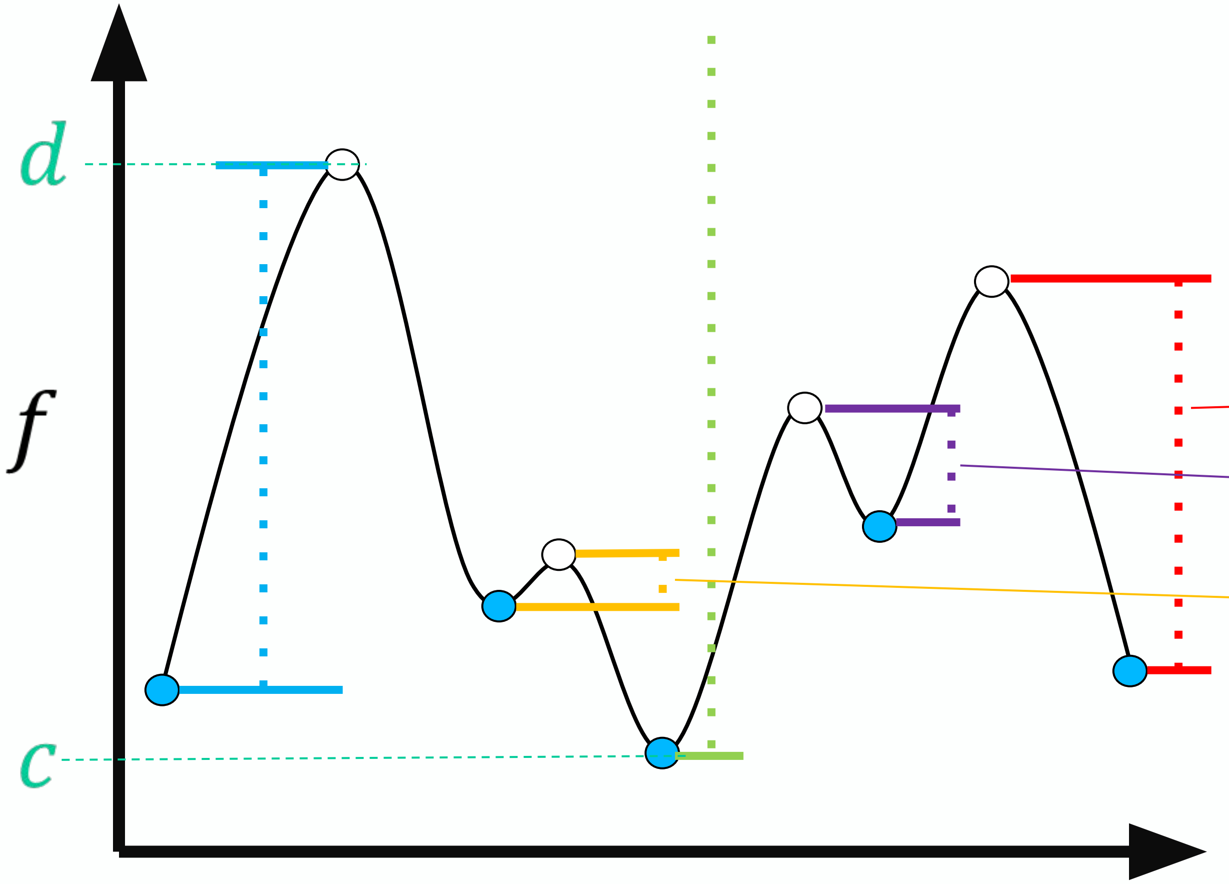
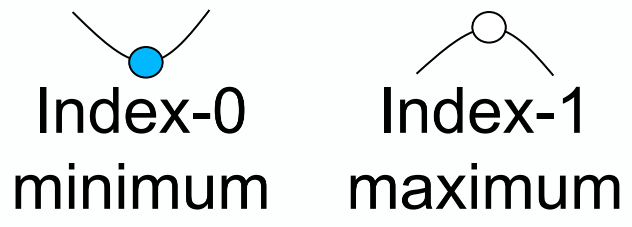
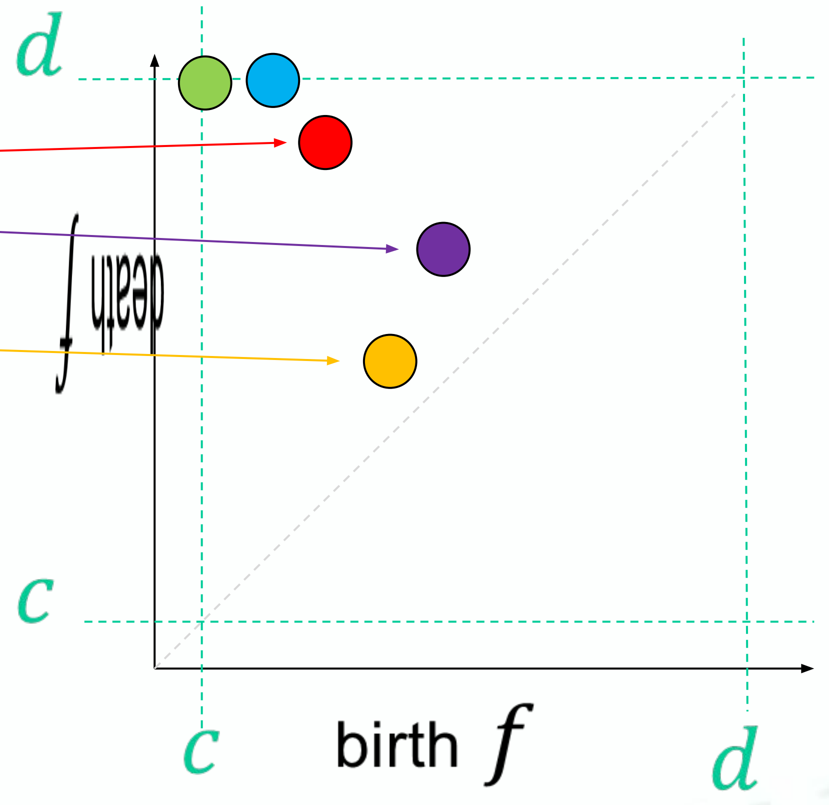
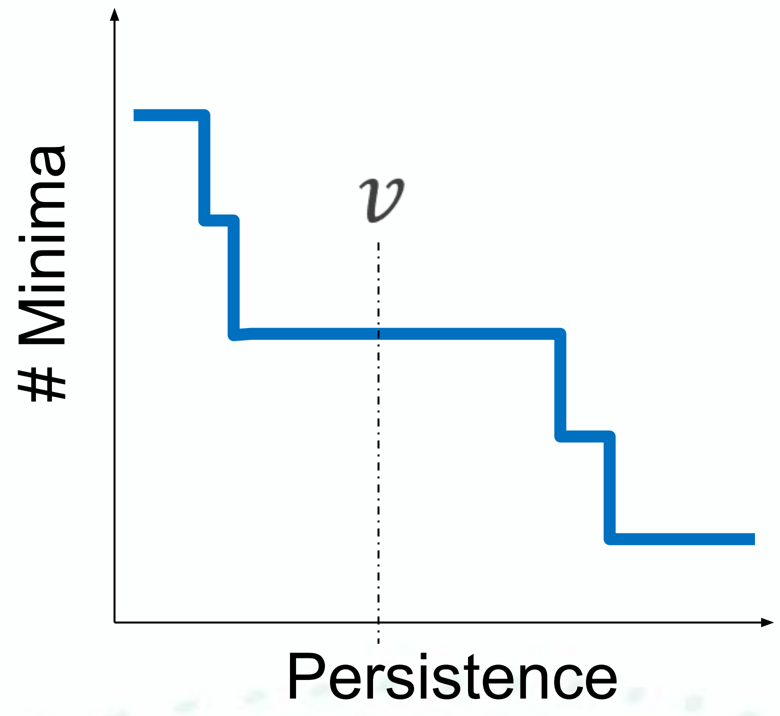




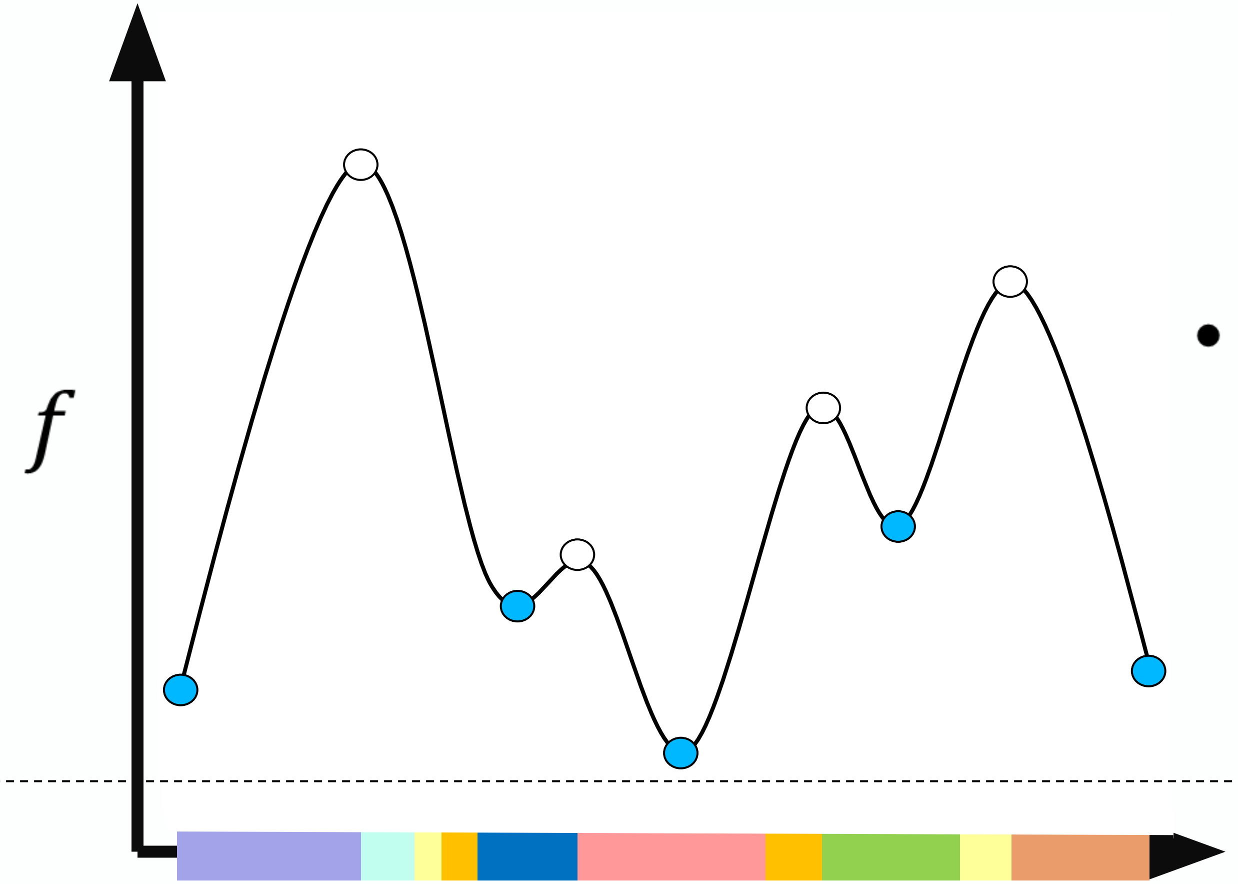


Persistence diagram



Persistence Plot

Features of a 1-dimensional function



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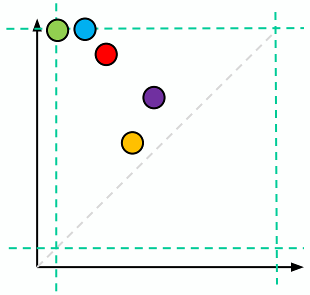
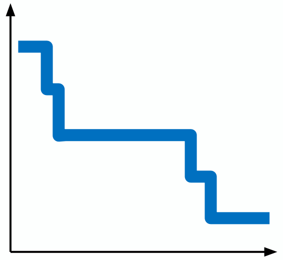
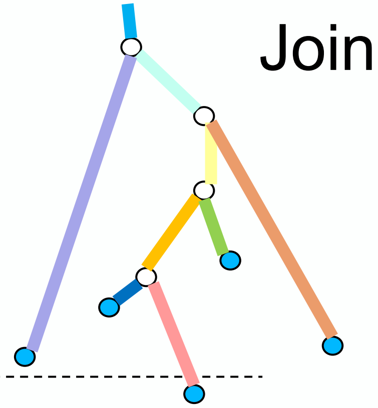
Index-0 minimum

Index-1 maximum





- Components existing during the filtration



Join tree

segmentation

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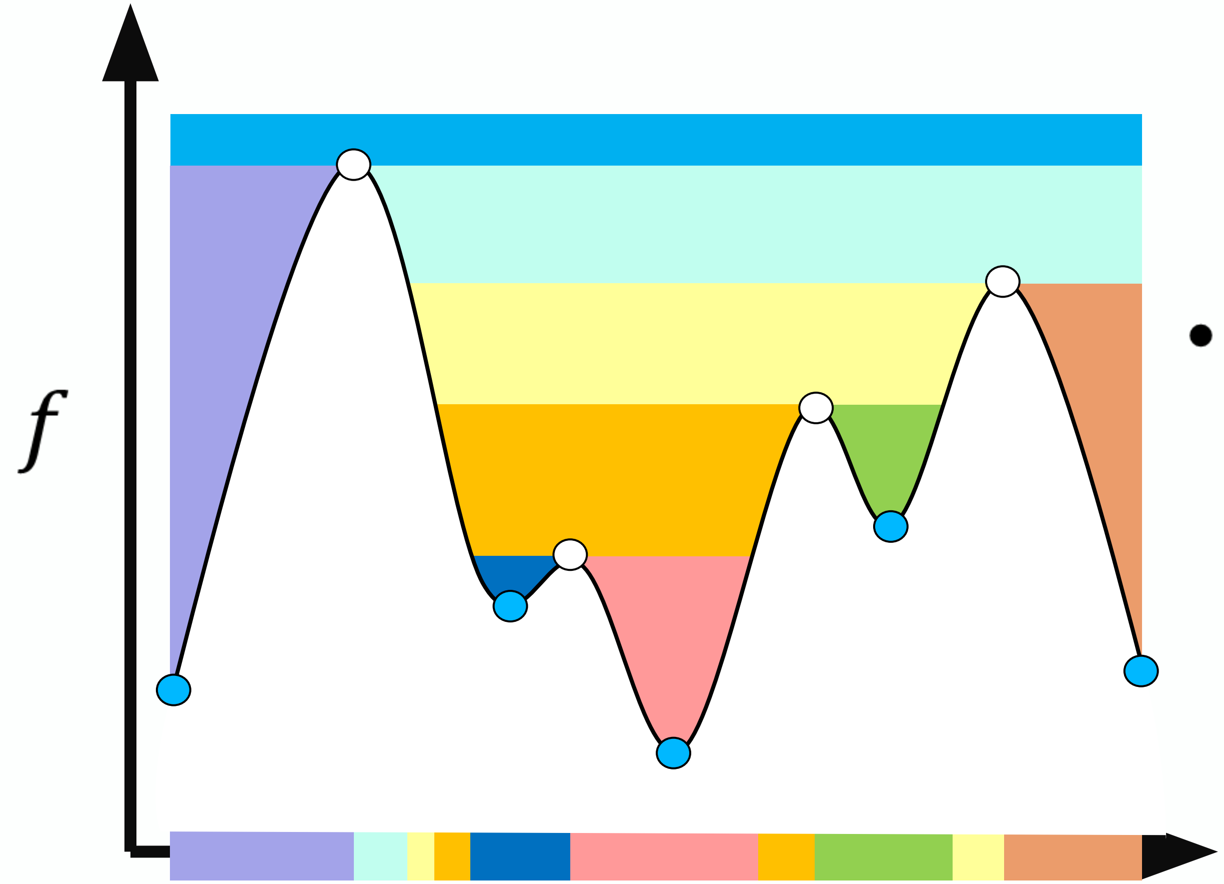

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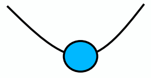

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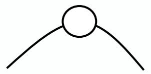

Index-1
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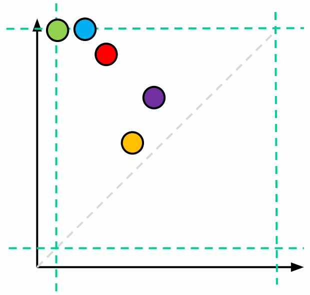

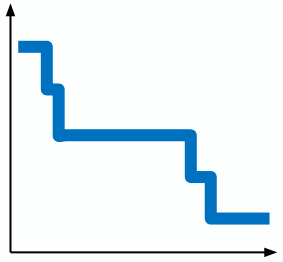

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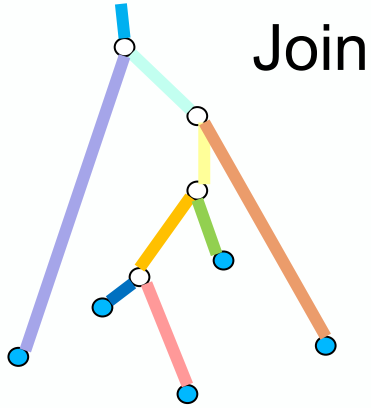

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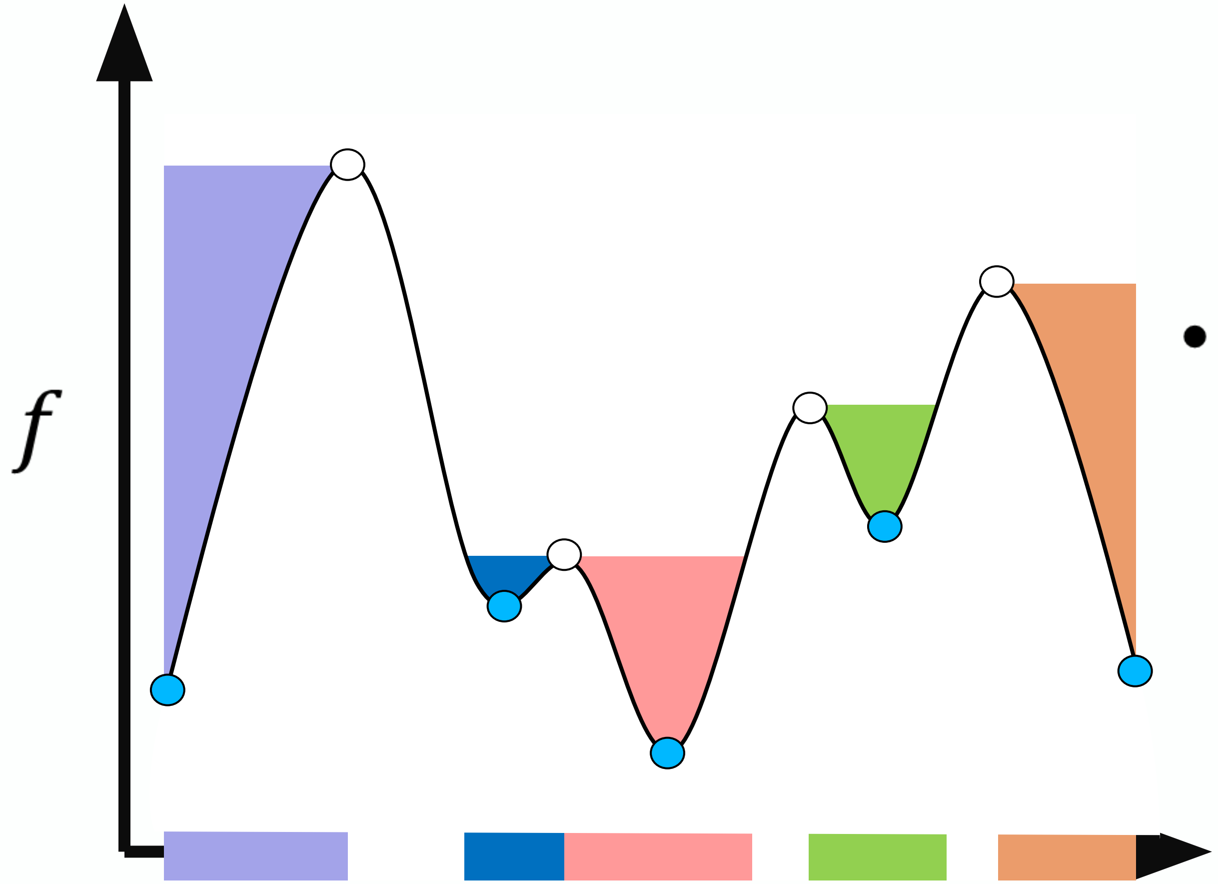

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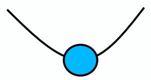

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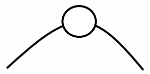

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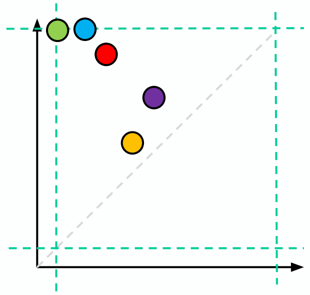

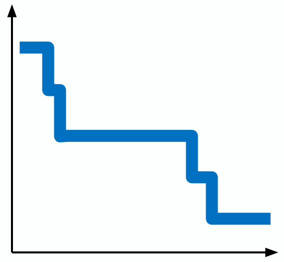

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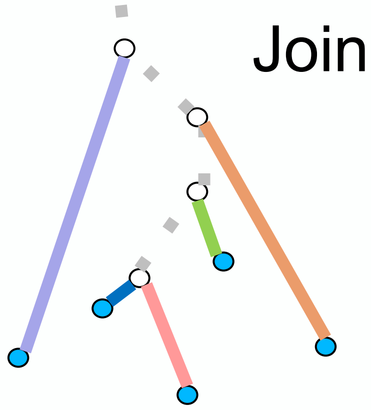

Join tree

“local valley”, “Extremal regions”

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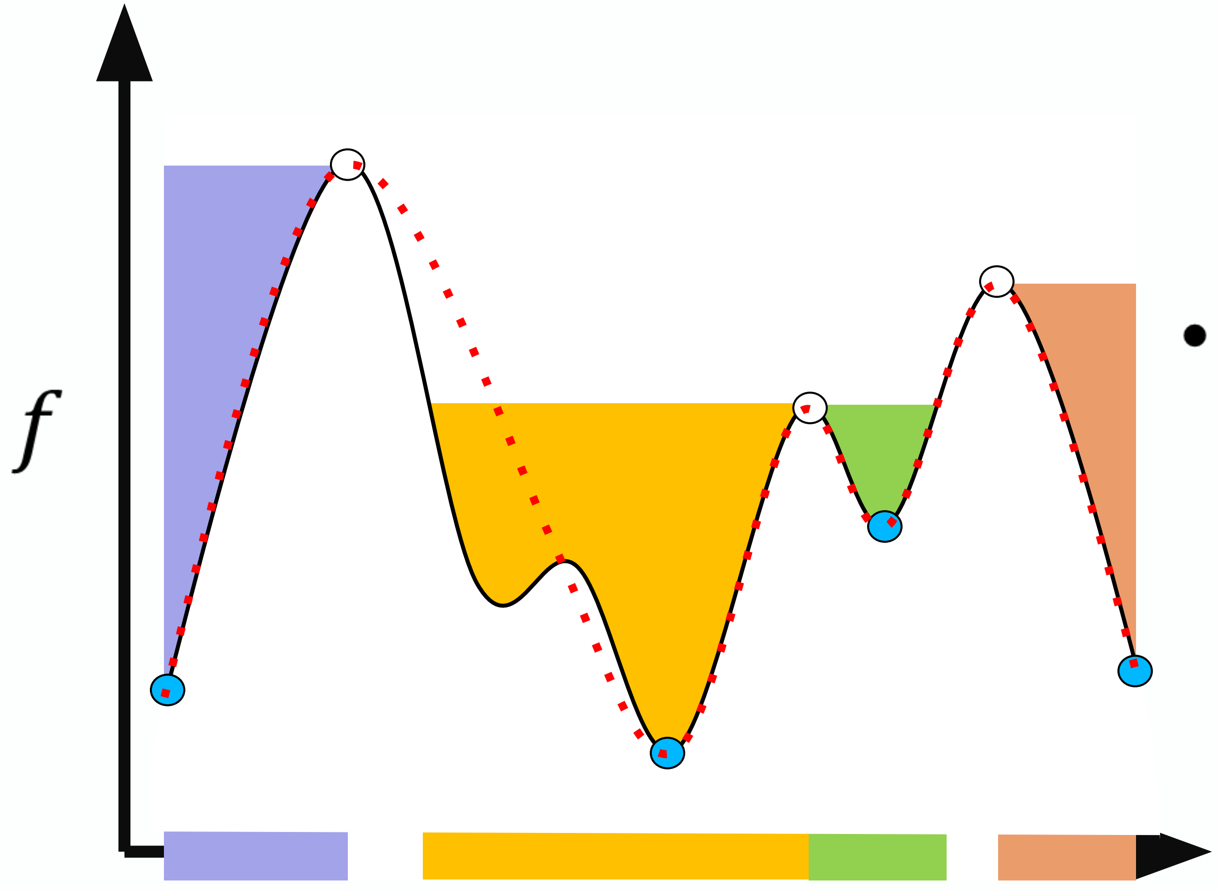

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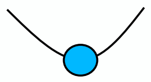

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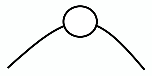

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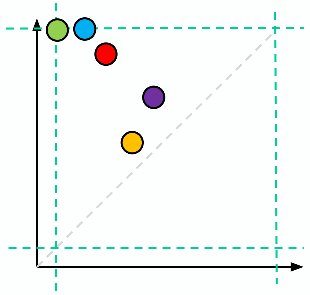

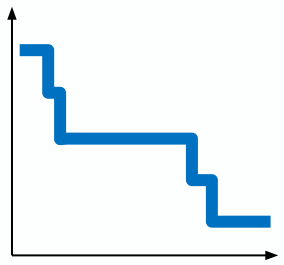

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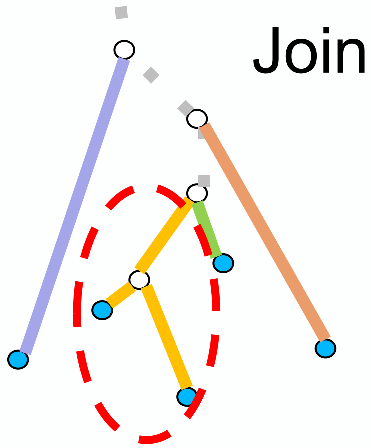

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"local valley", "Extremal regions"

"at persistence x"

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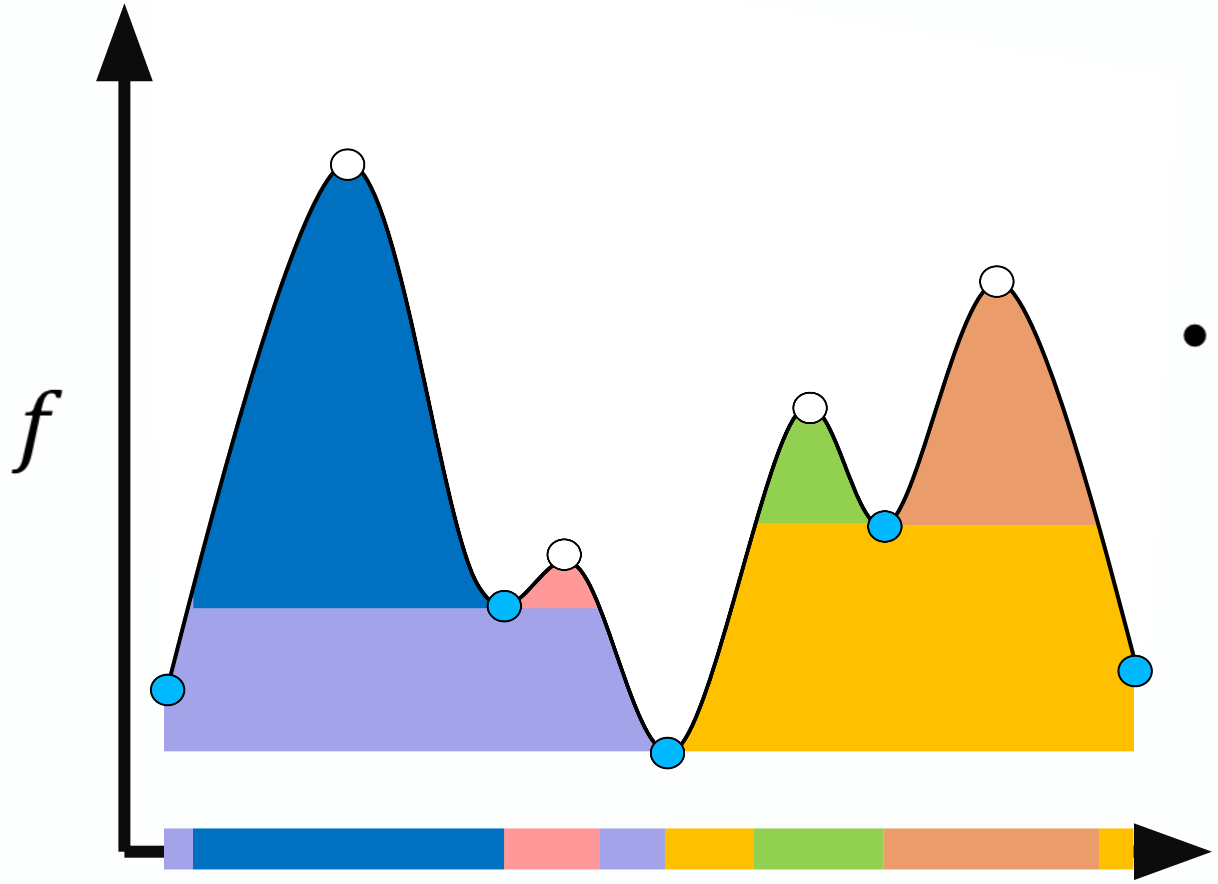

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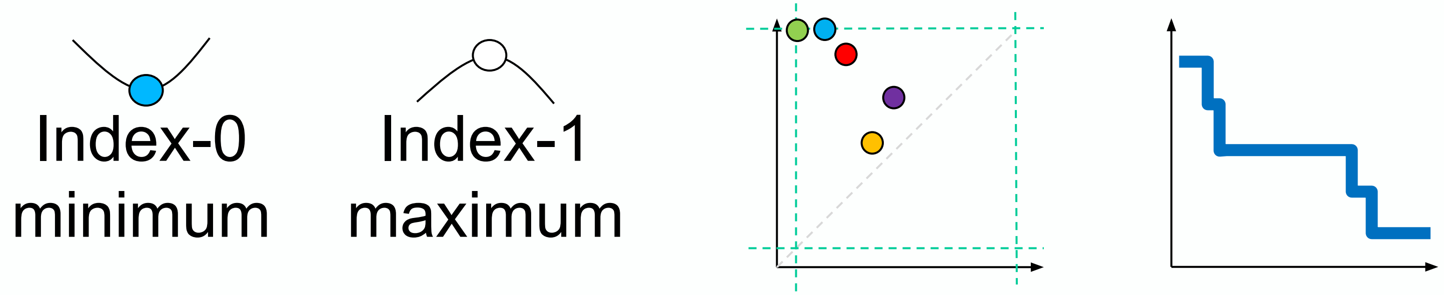

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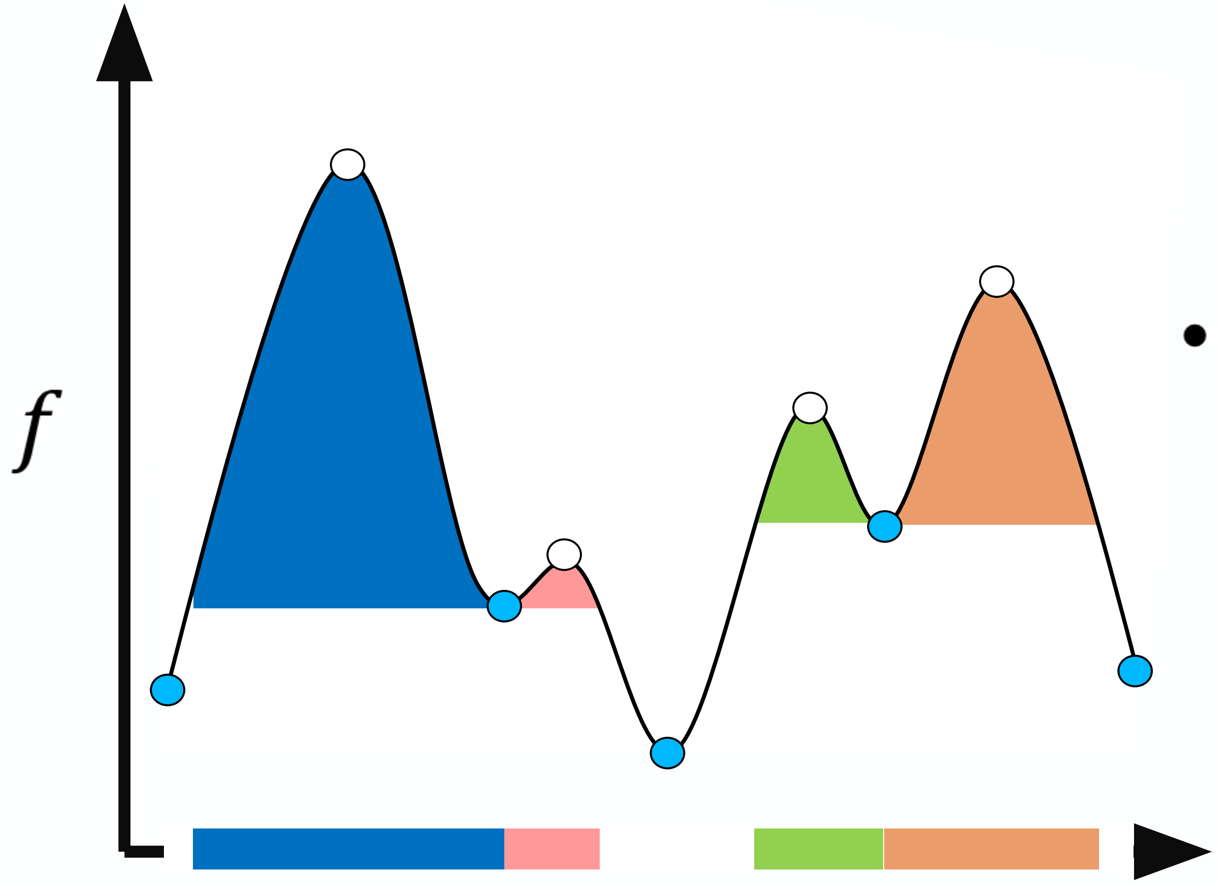

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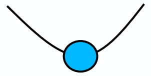

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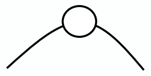

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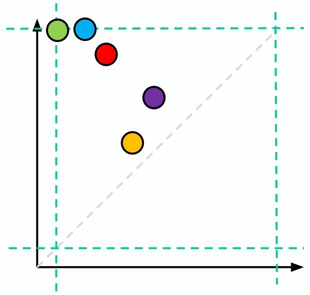

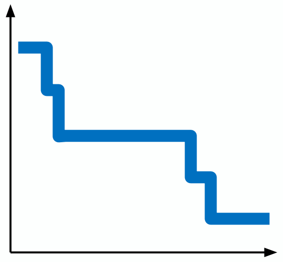

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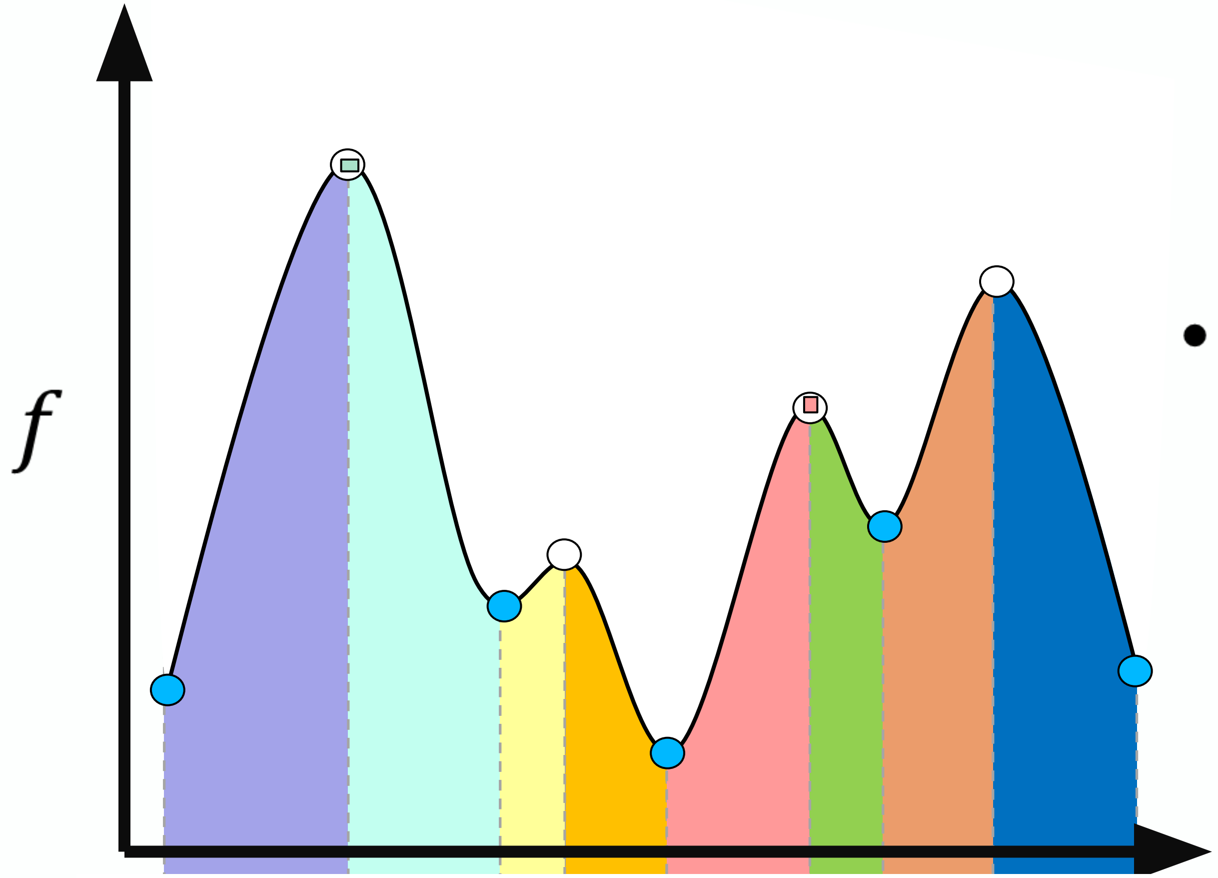

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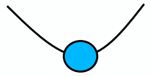

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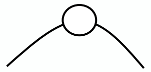

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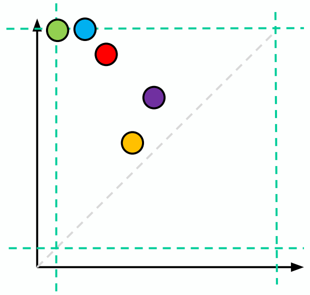

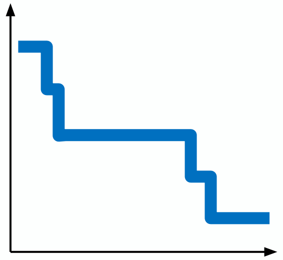

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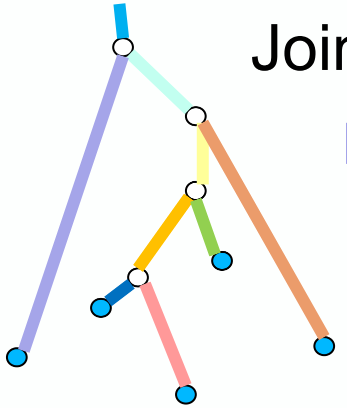
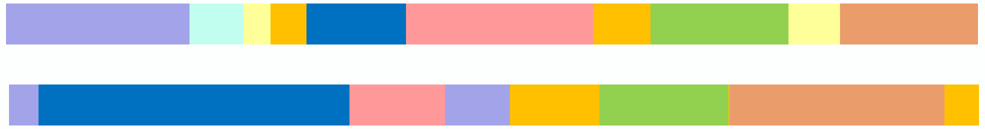
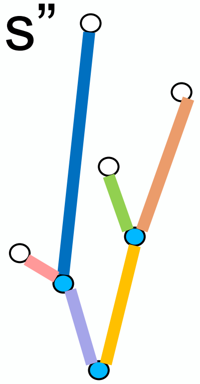


Join tree "local valley", "Extremal regions"

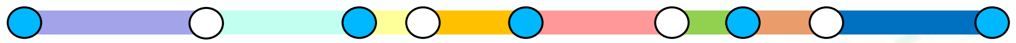


"local peak" Split tree



- Locally monotonic regions

Morse-Smale Complex nodes/arcs



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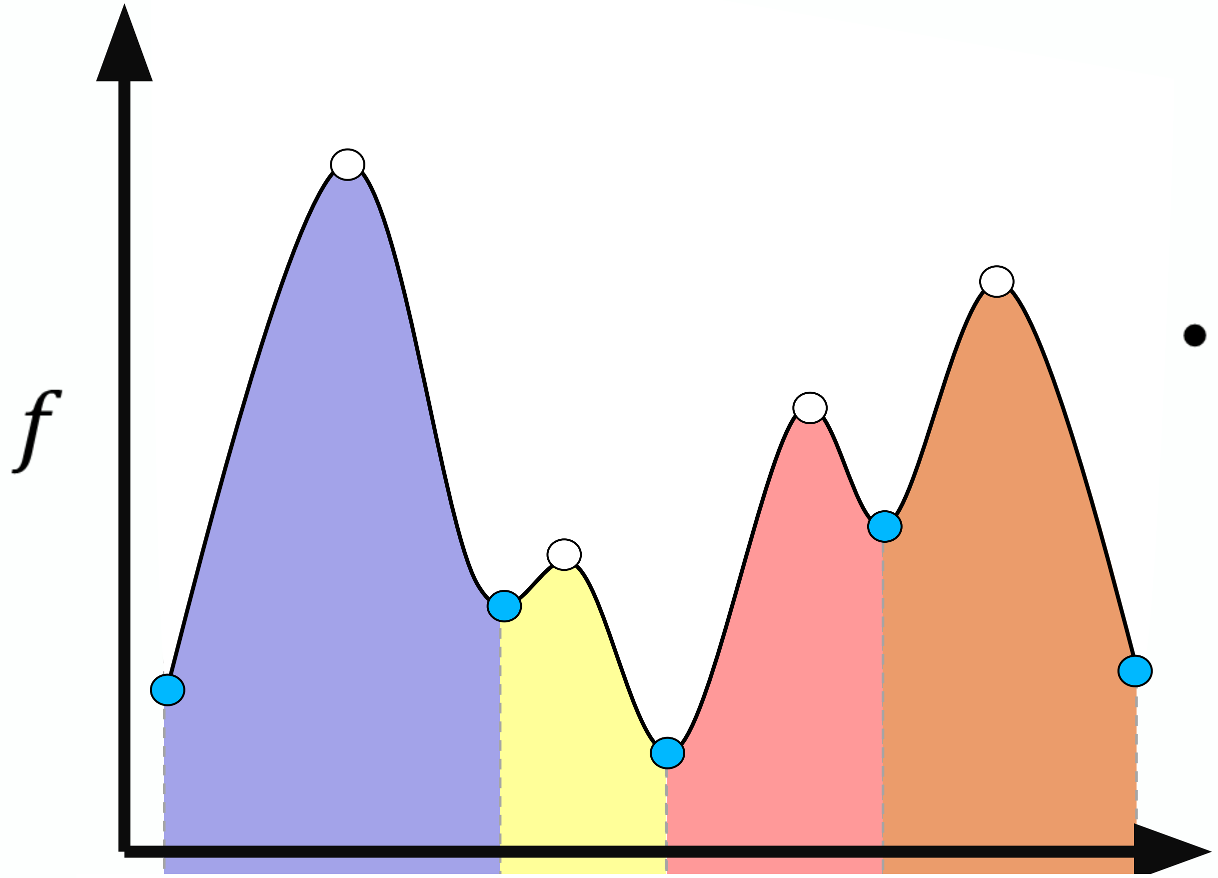
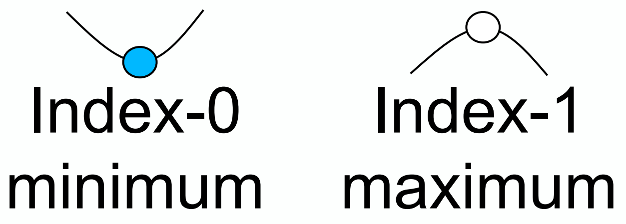
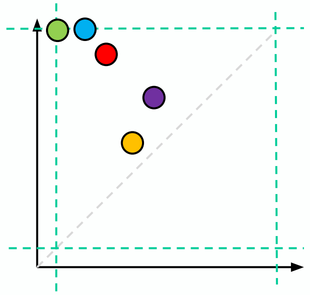
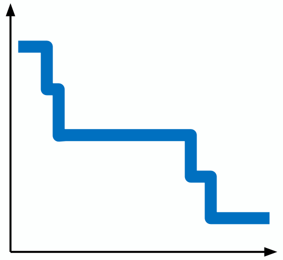
Features of a 1-dimensional function



- Critical points where $\nabla f = 0$

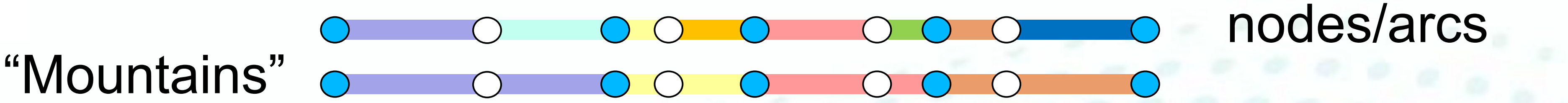




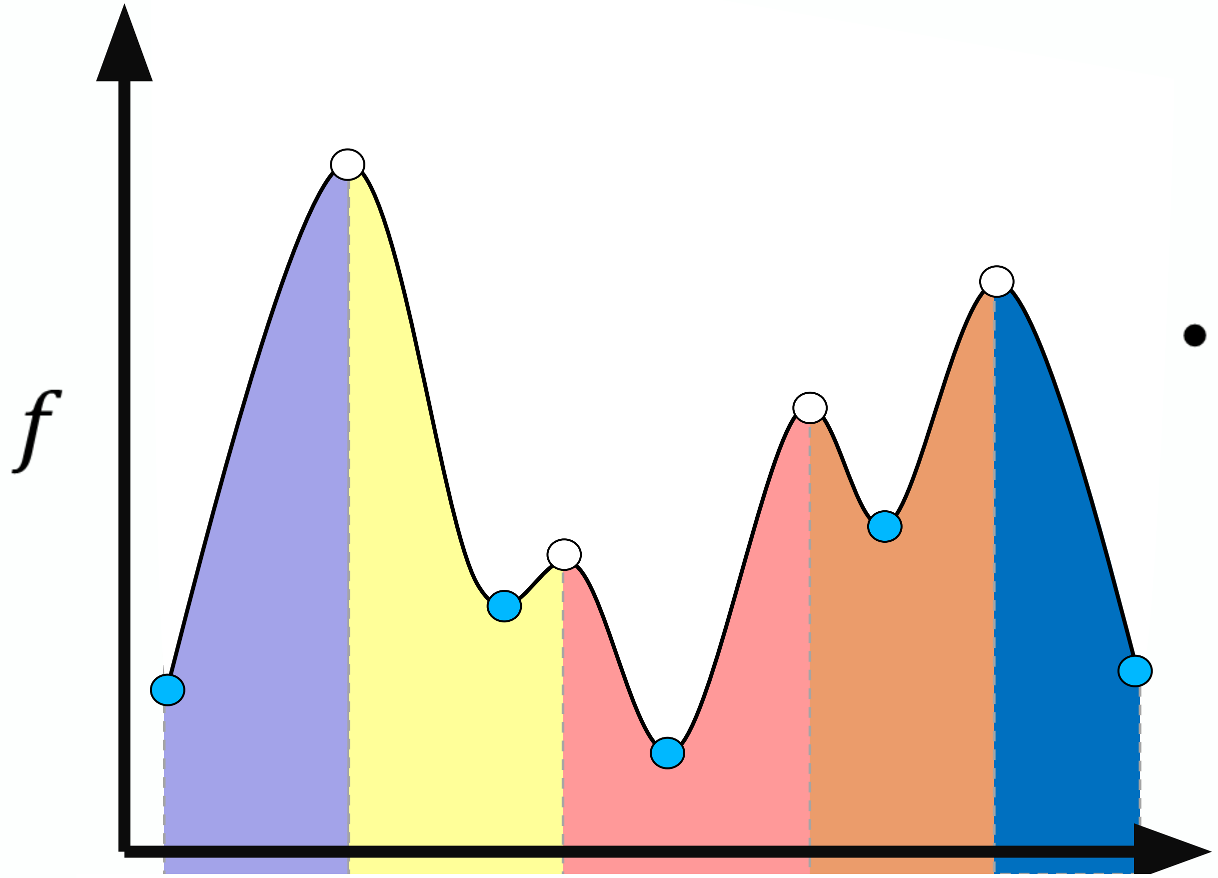


- Components existing during the filtration

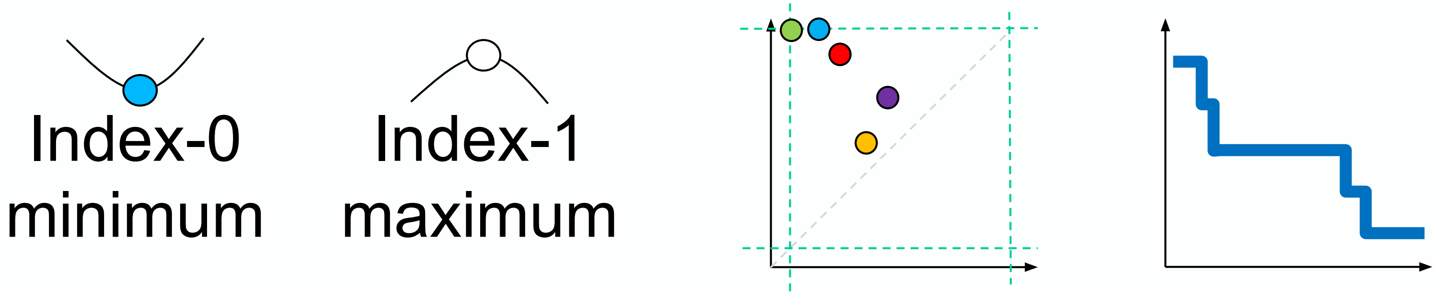
- Locally monotonic regions



Features of a 1-dimensional function

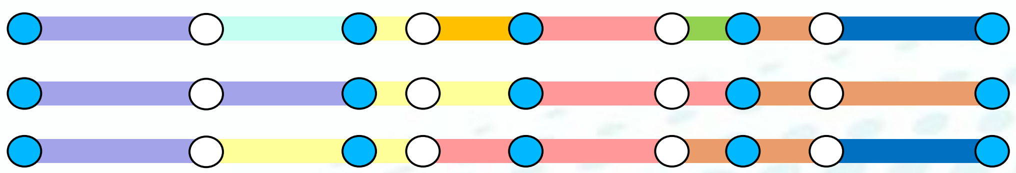


- Critical points where $\nabla f = 0$



- Components existing during the filtration

- Locally monotonic regions



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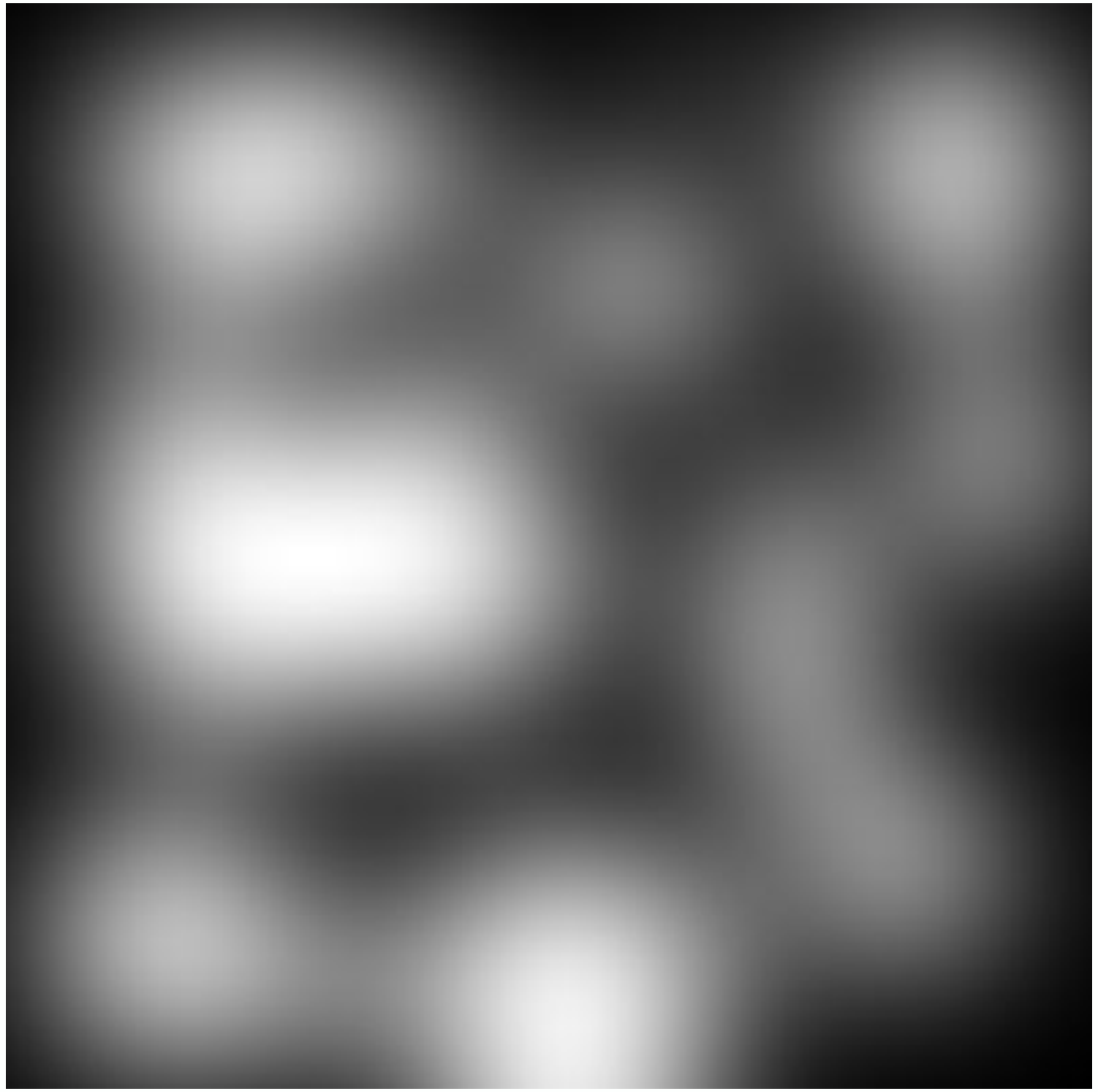
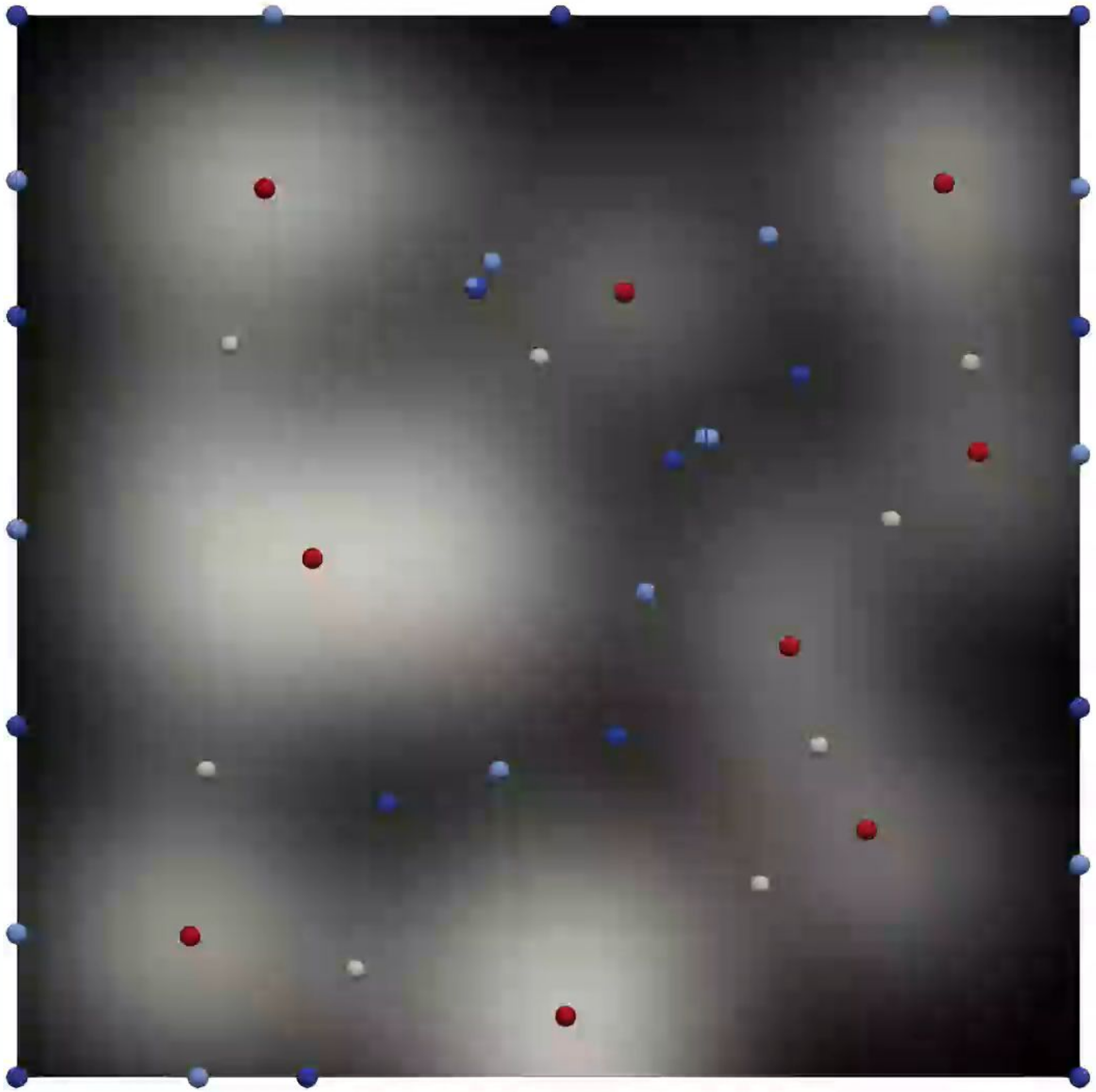
Features of a 2-dimensional function

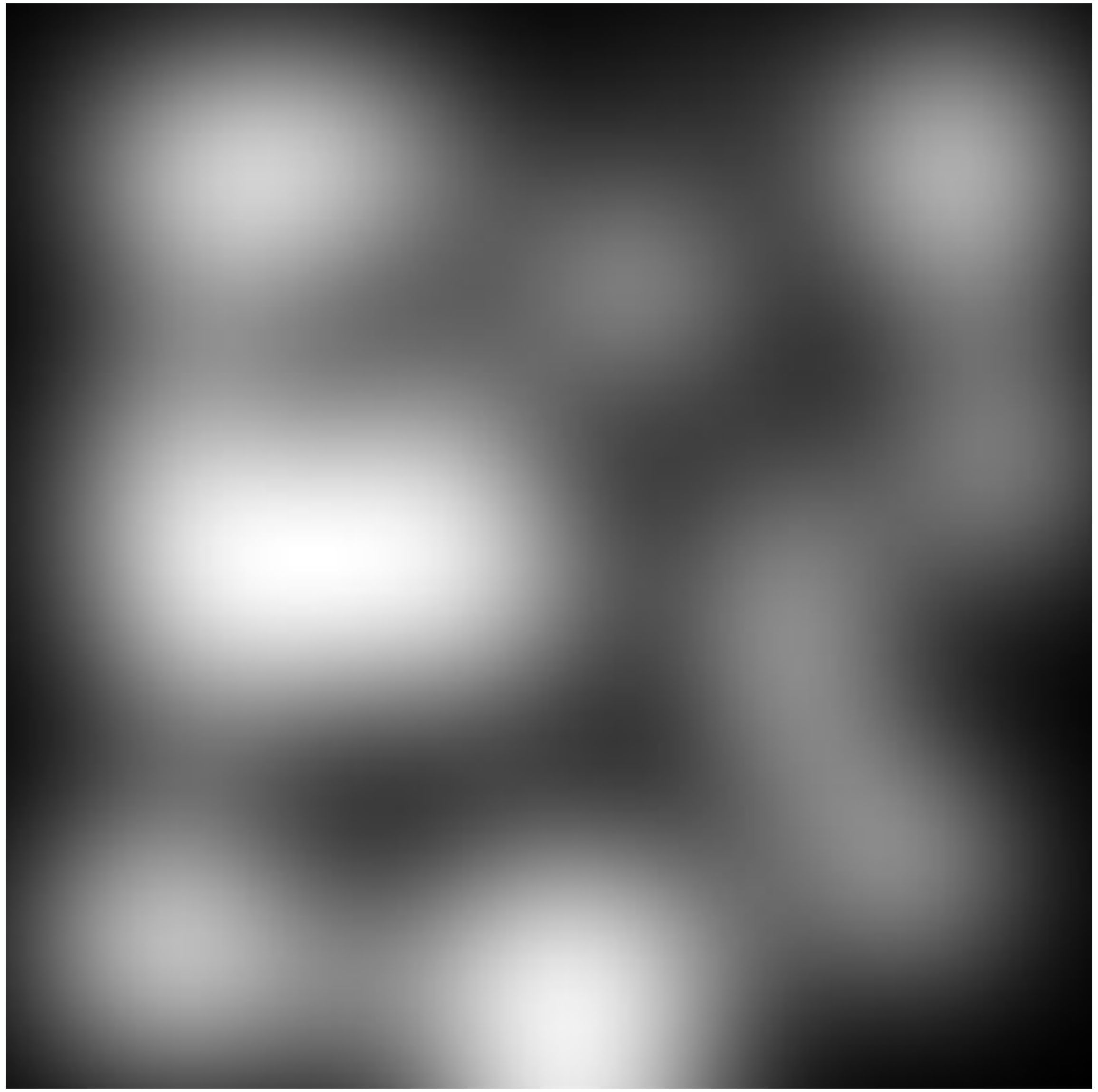


Grayscale rendering
Of 2d scalar function

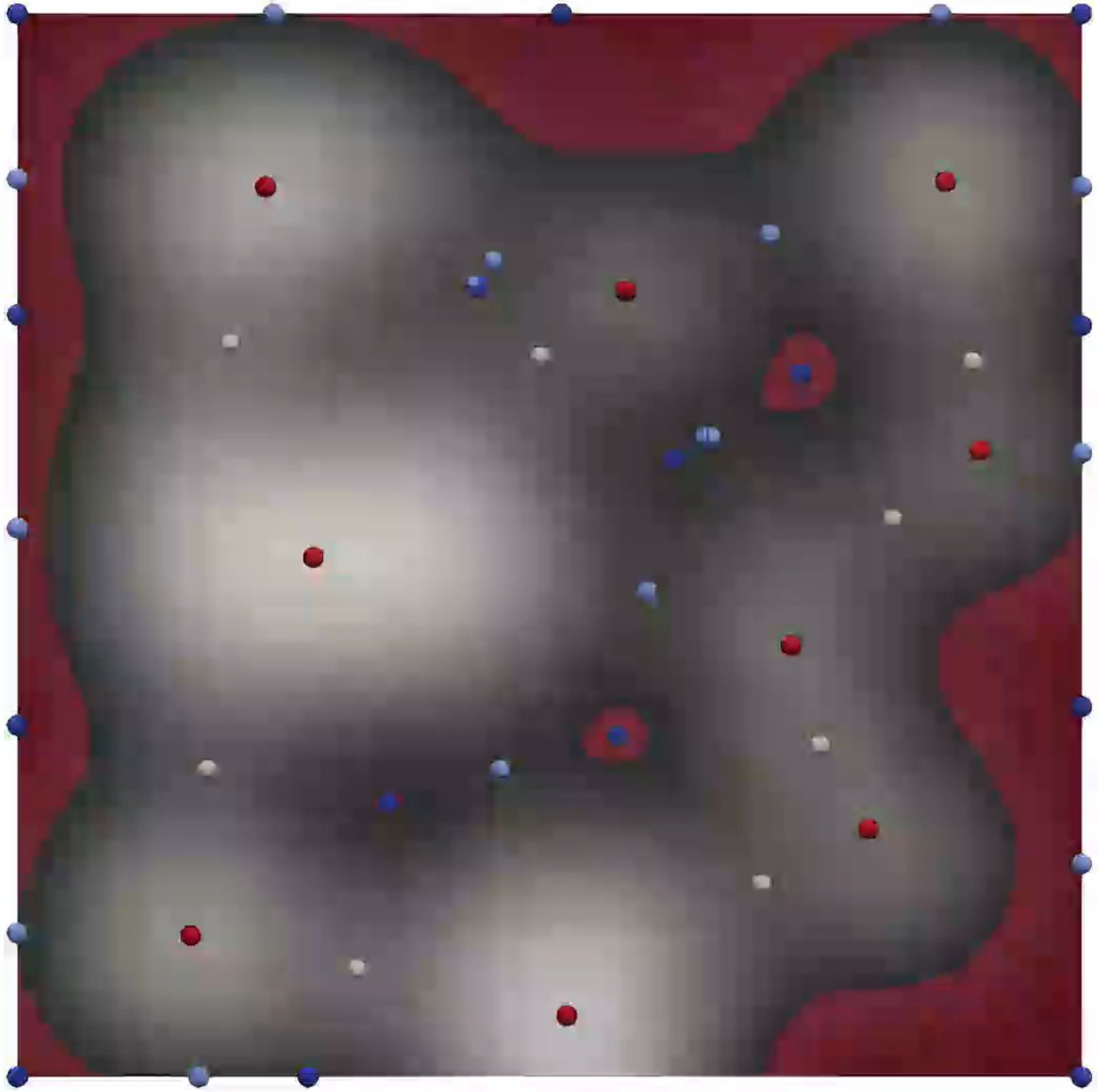


$$f^{-1}((-\infty, v])$$

Features of a 2-dimensional function

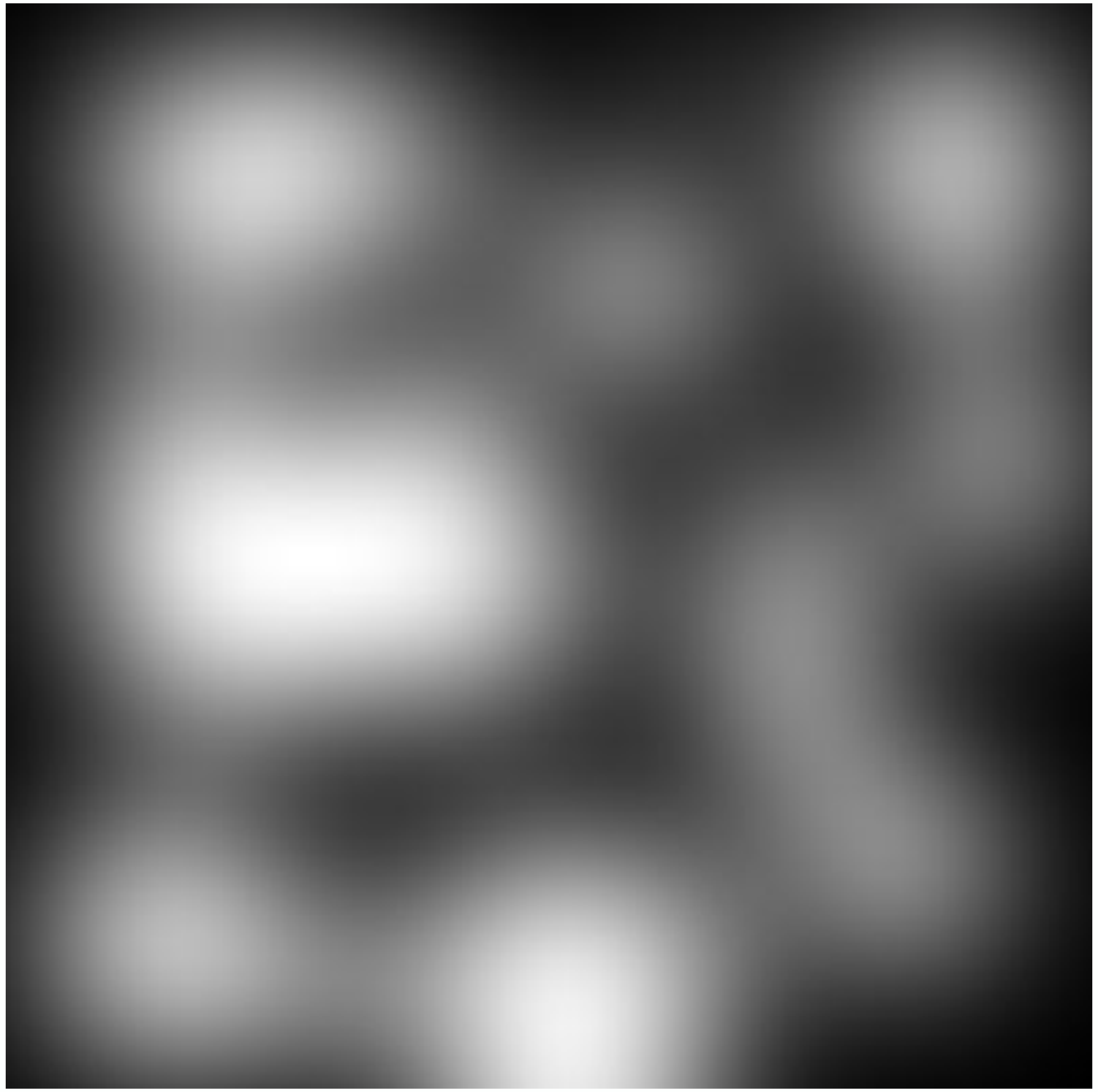


Grayscale rendering
Of 2d scalar function

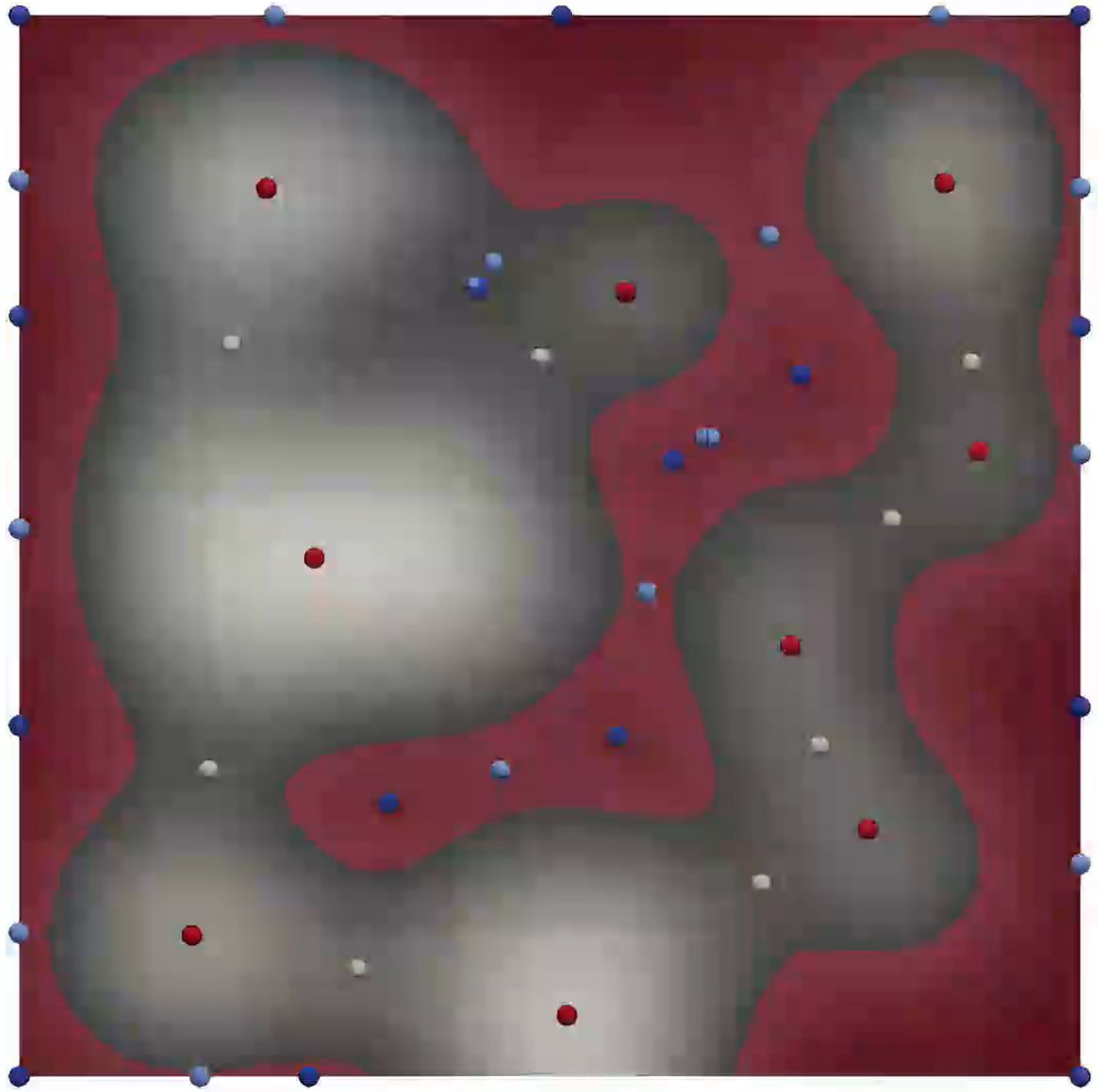


$f^{-1}((-\infty, v])$

Features of a 2-dimensional function

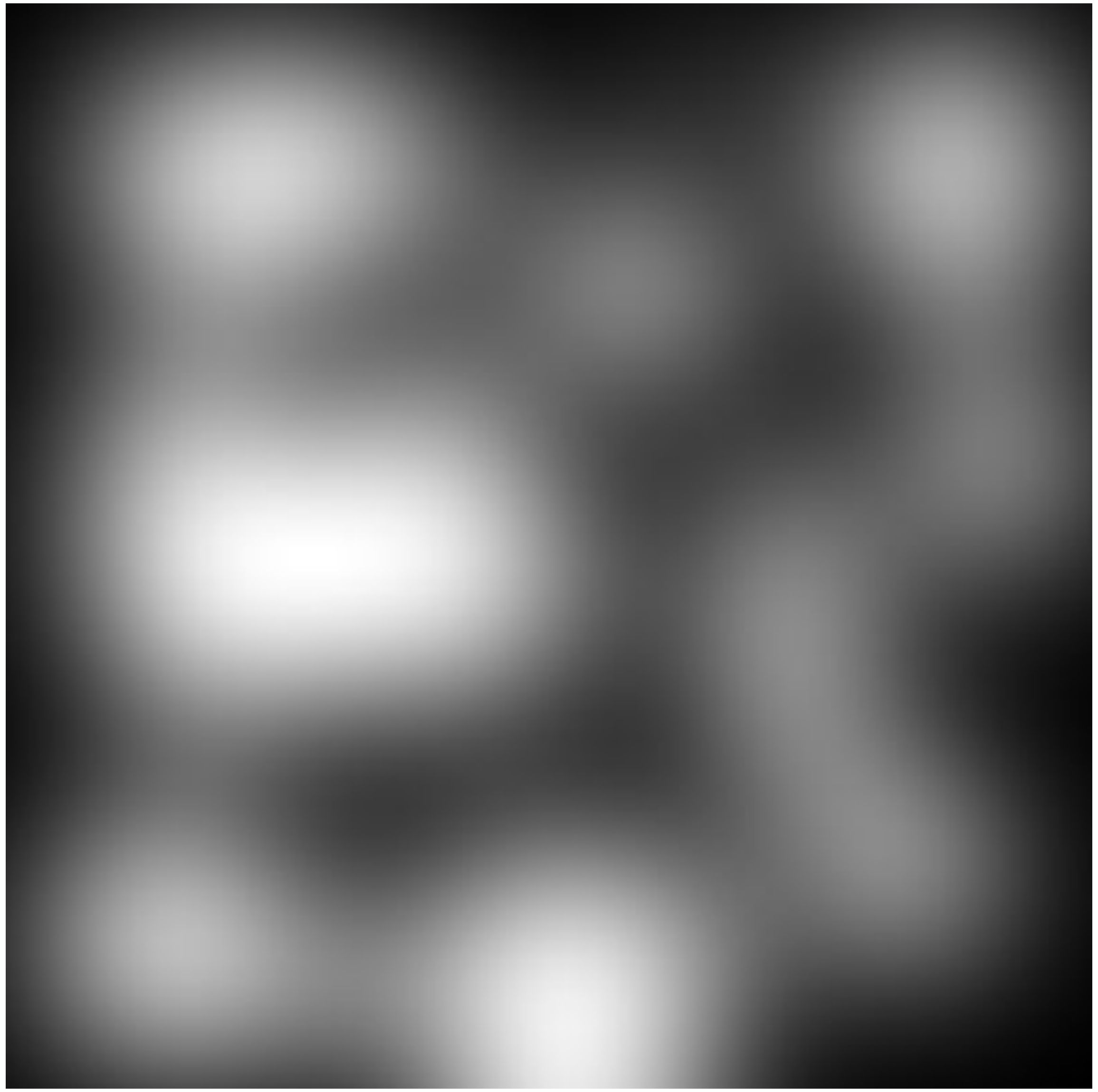


Grayscale rendering
Of 2d scalar function

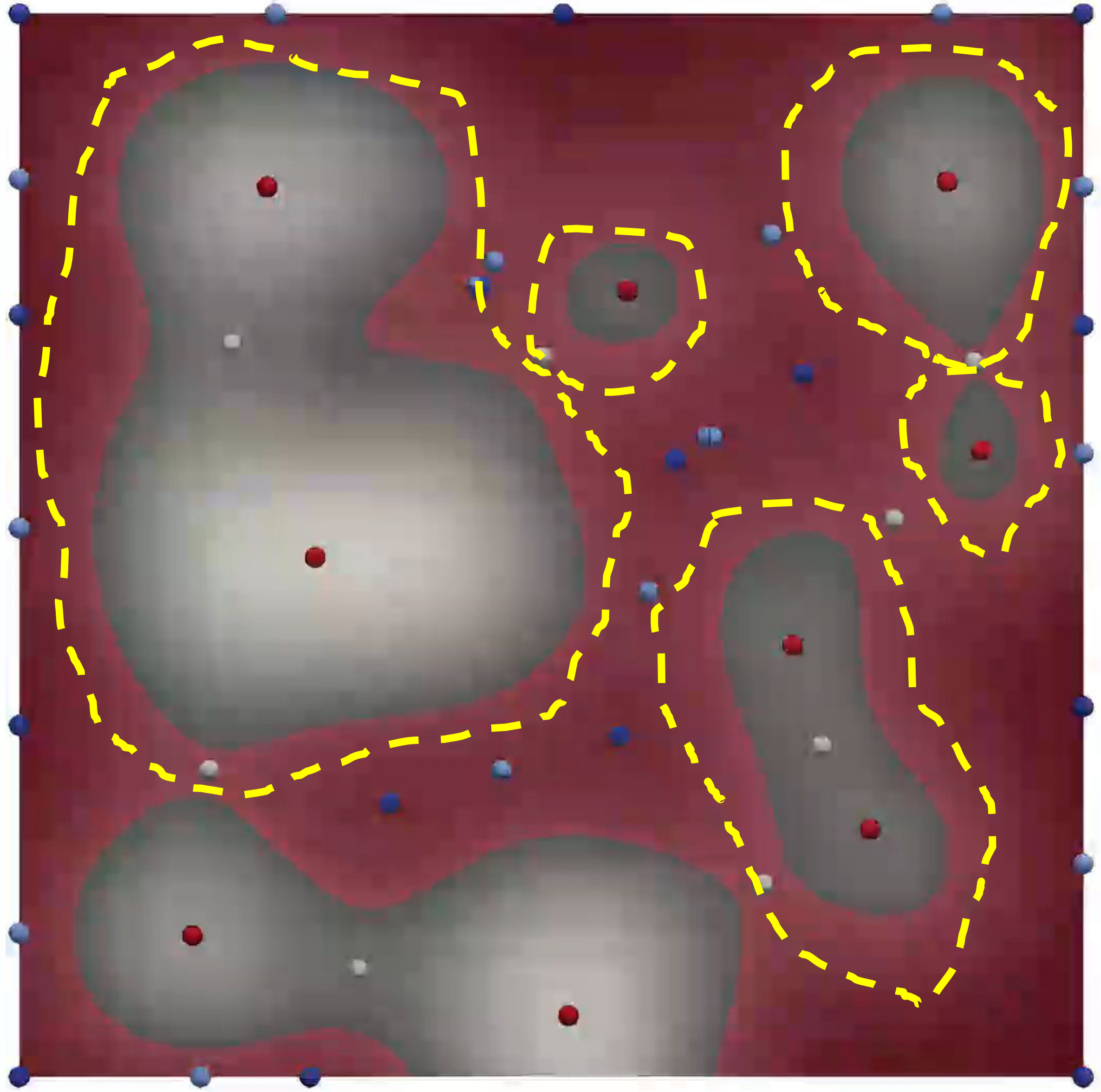


$f^{-1}((-\infty, v])$

Features of a 2-dimensional function

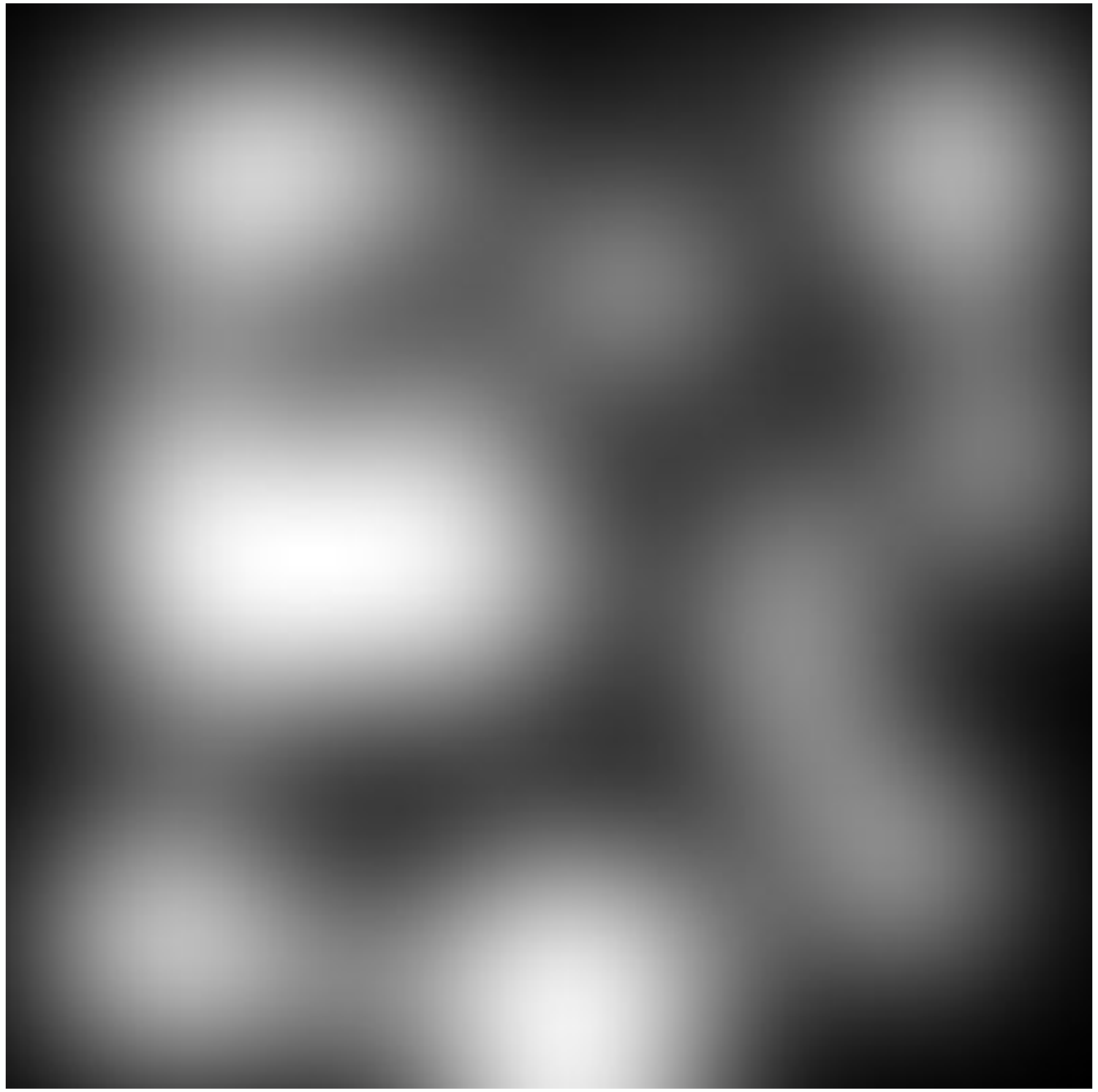


Grayscale rendering
Of 2d scalar function

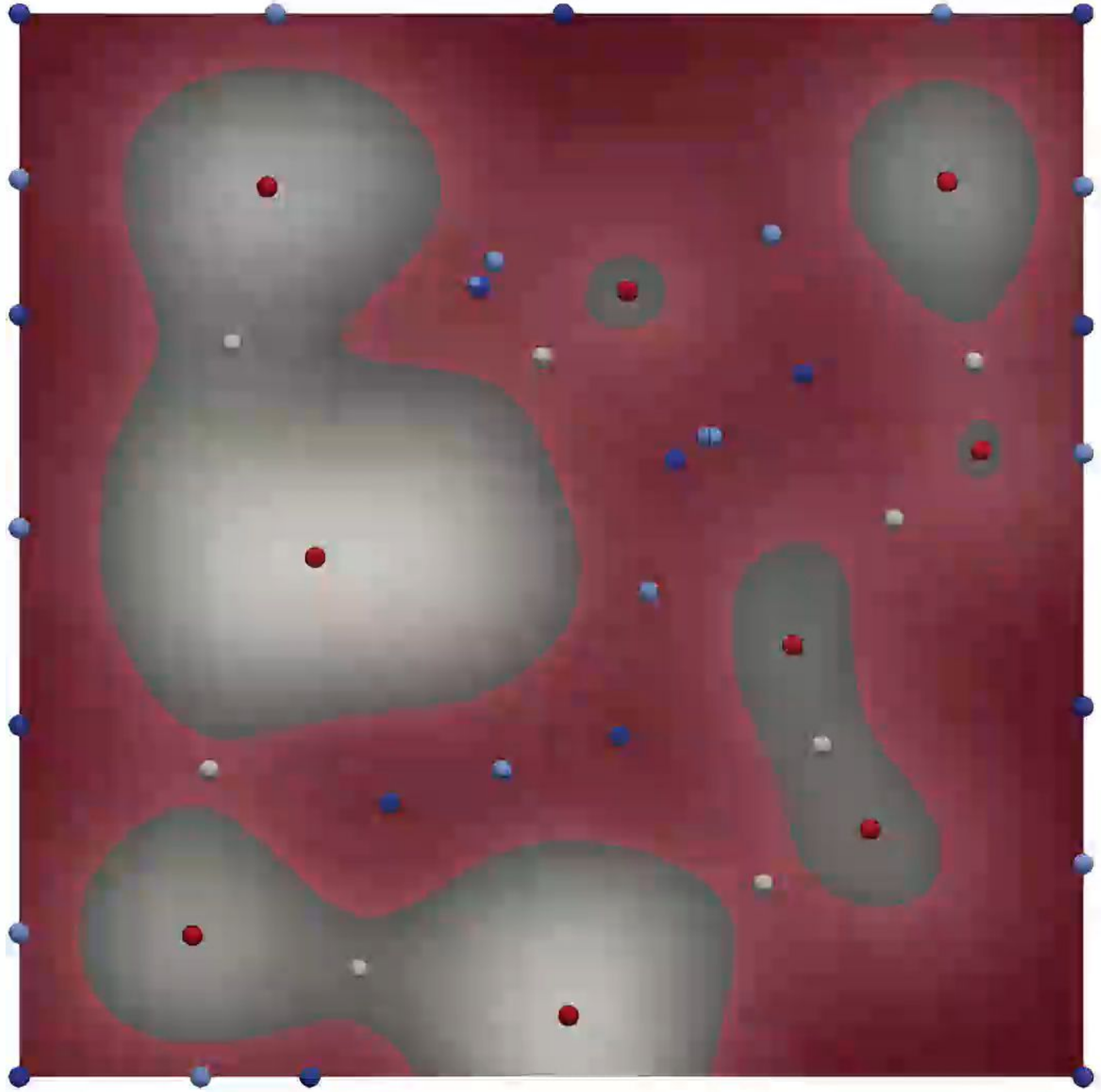


$f^{-1}((-\infty, v])$

Features of a 2-dimensional function



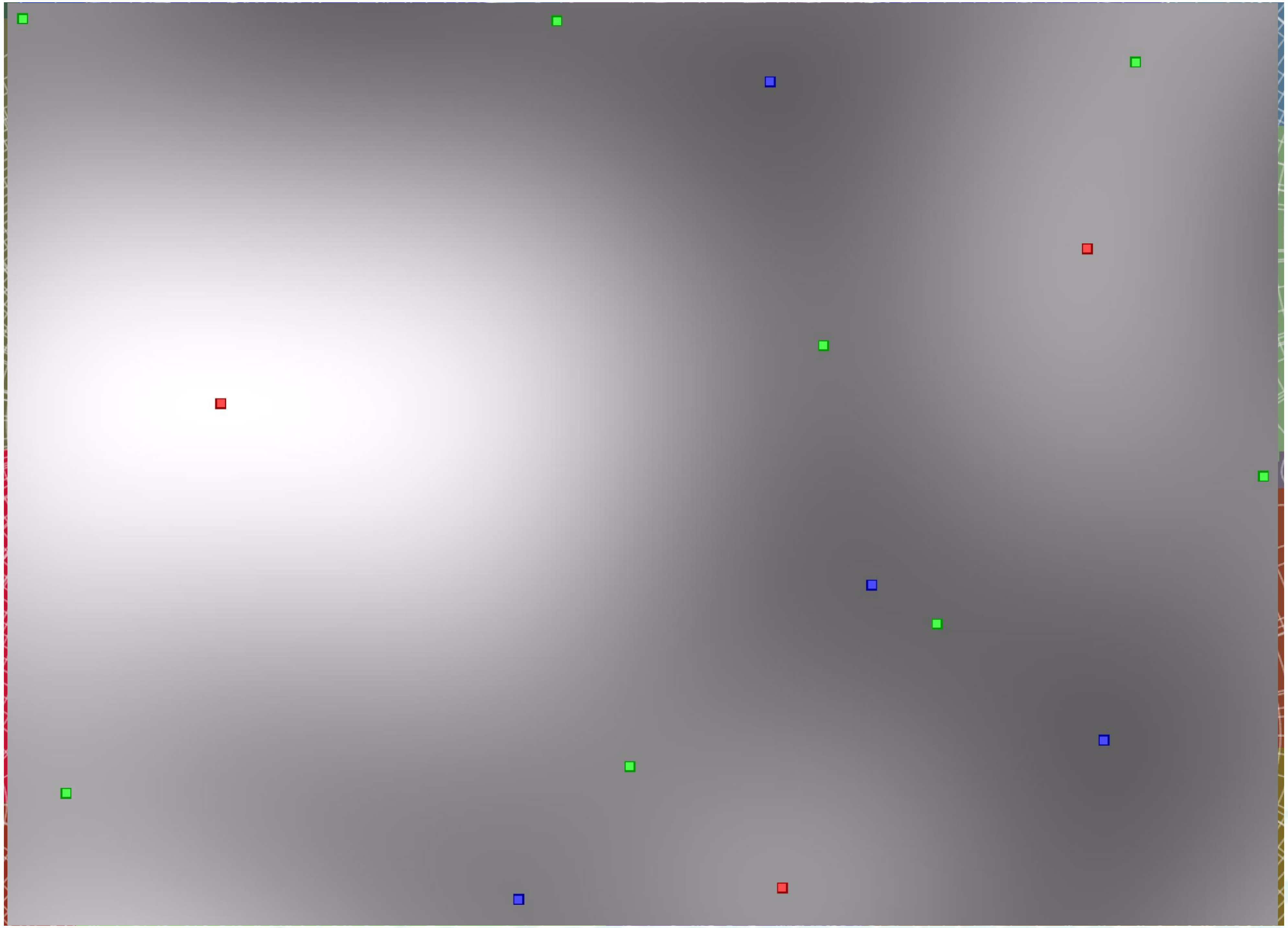
Grayscale rendering
Of 2d scalar function



$f^{-1}((-\infty, v])$

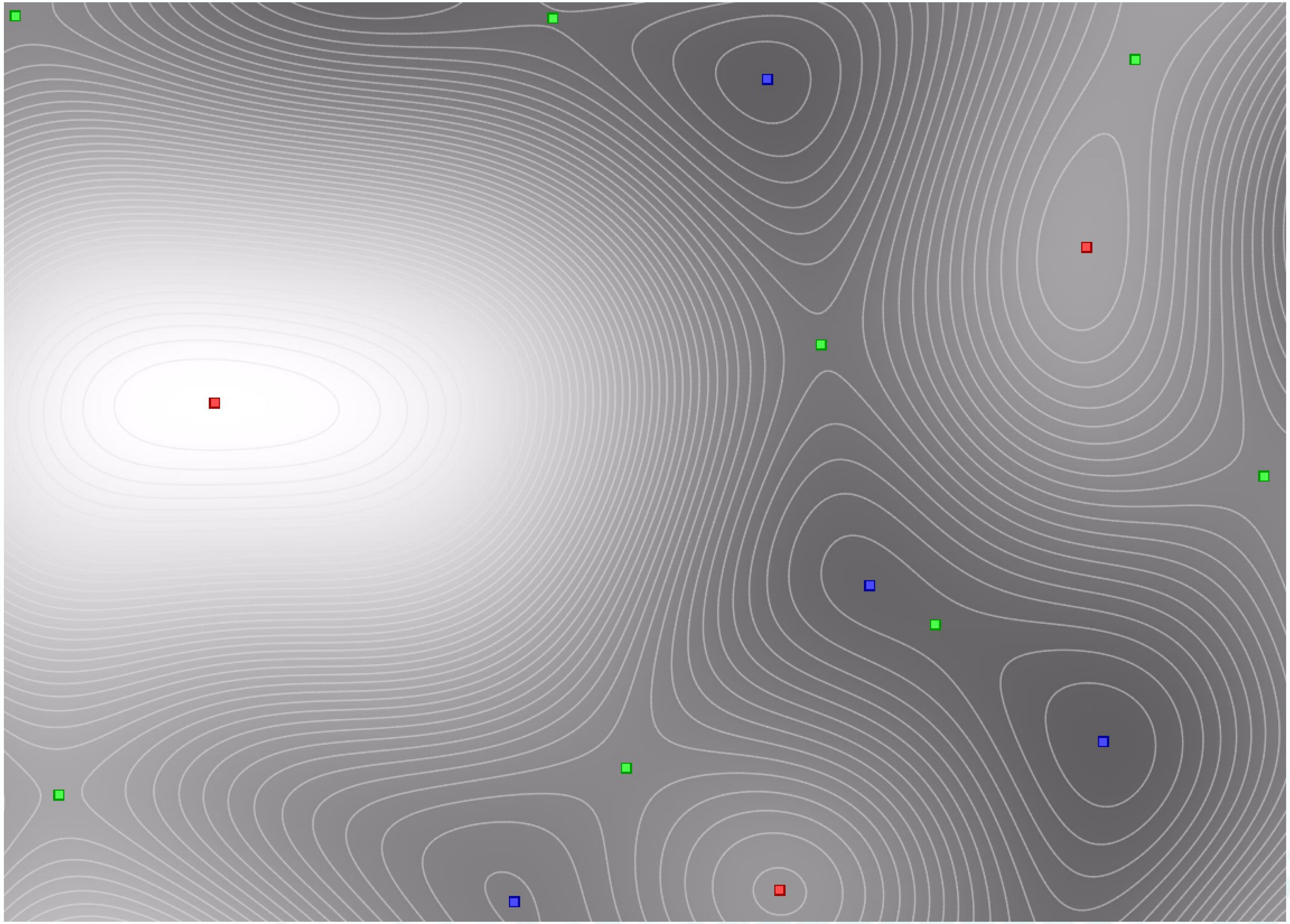
Features of a 2-dimensional function

"Monotonicity" = pieces of contours grouped by integral lines



Features of a 2-dimensional function

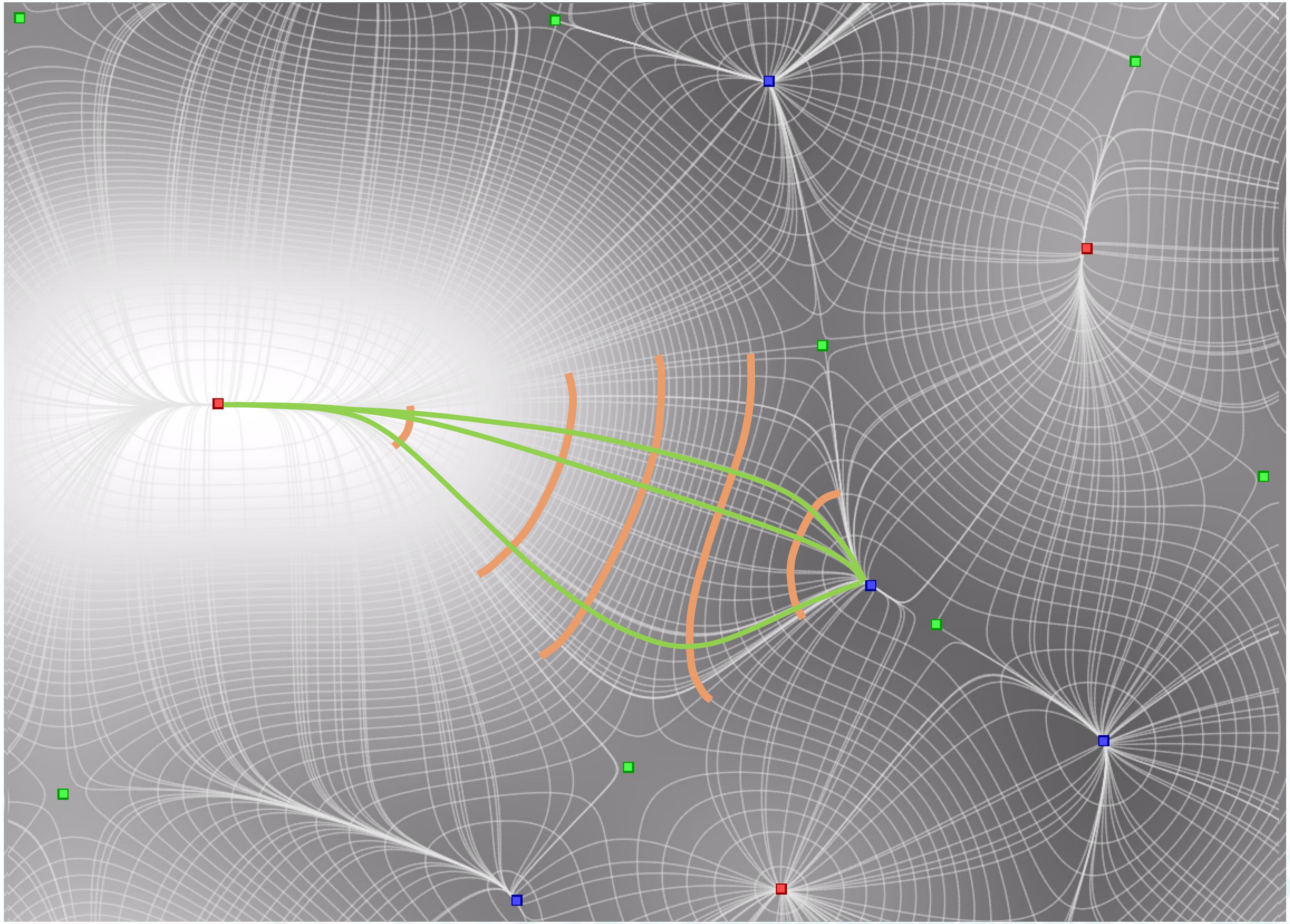
"Monotonicity" = pieces of contours grouped by integral lines



Features of a 2-dimensional function

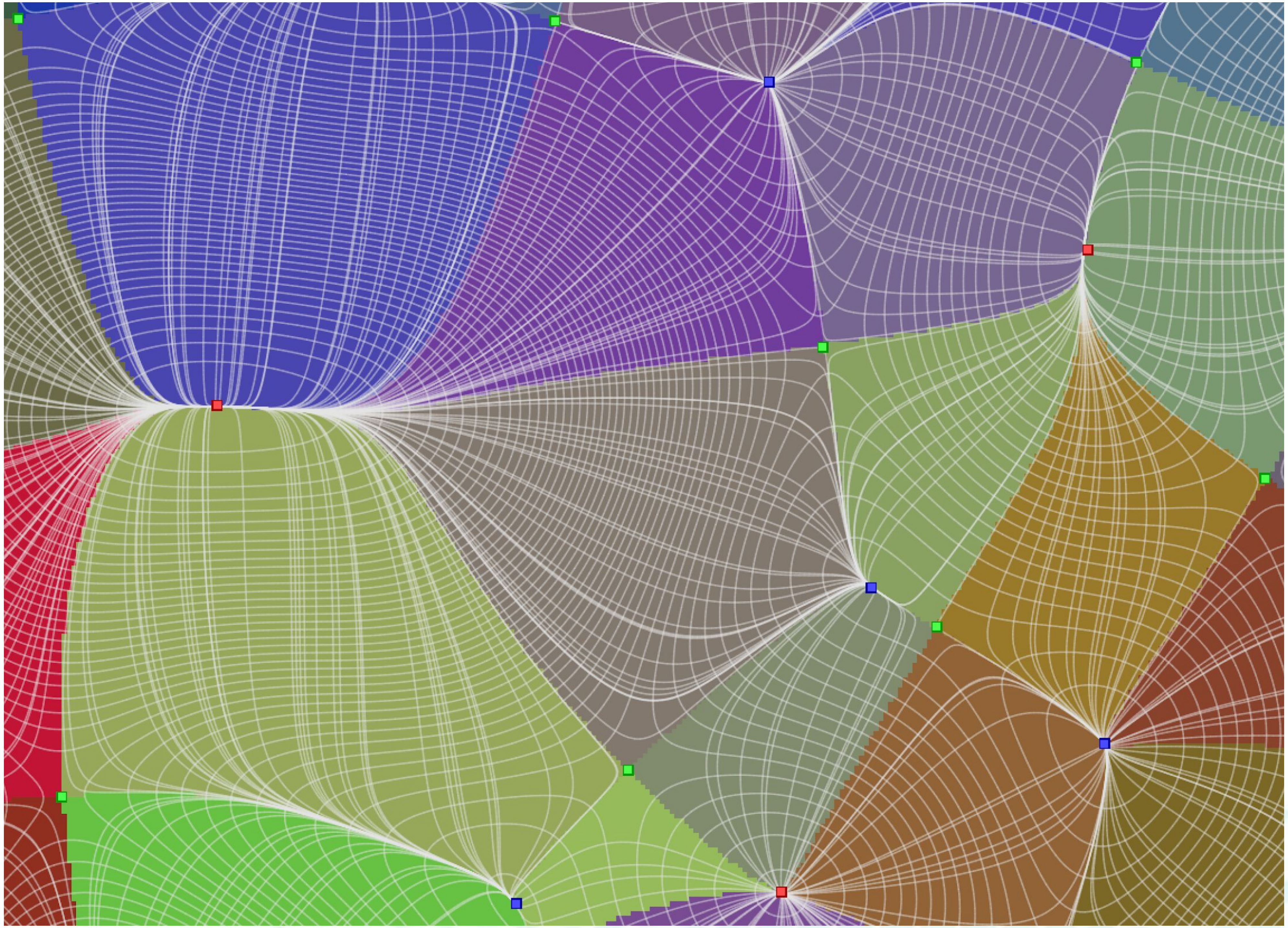
"Monotonicity" = pieces of contours grouped by integral lines

$$\frac{\partial}{\partial t} L(t) = \nabla f(L(t))$$



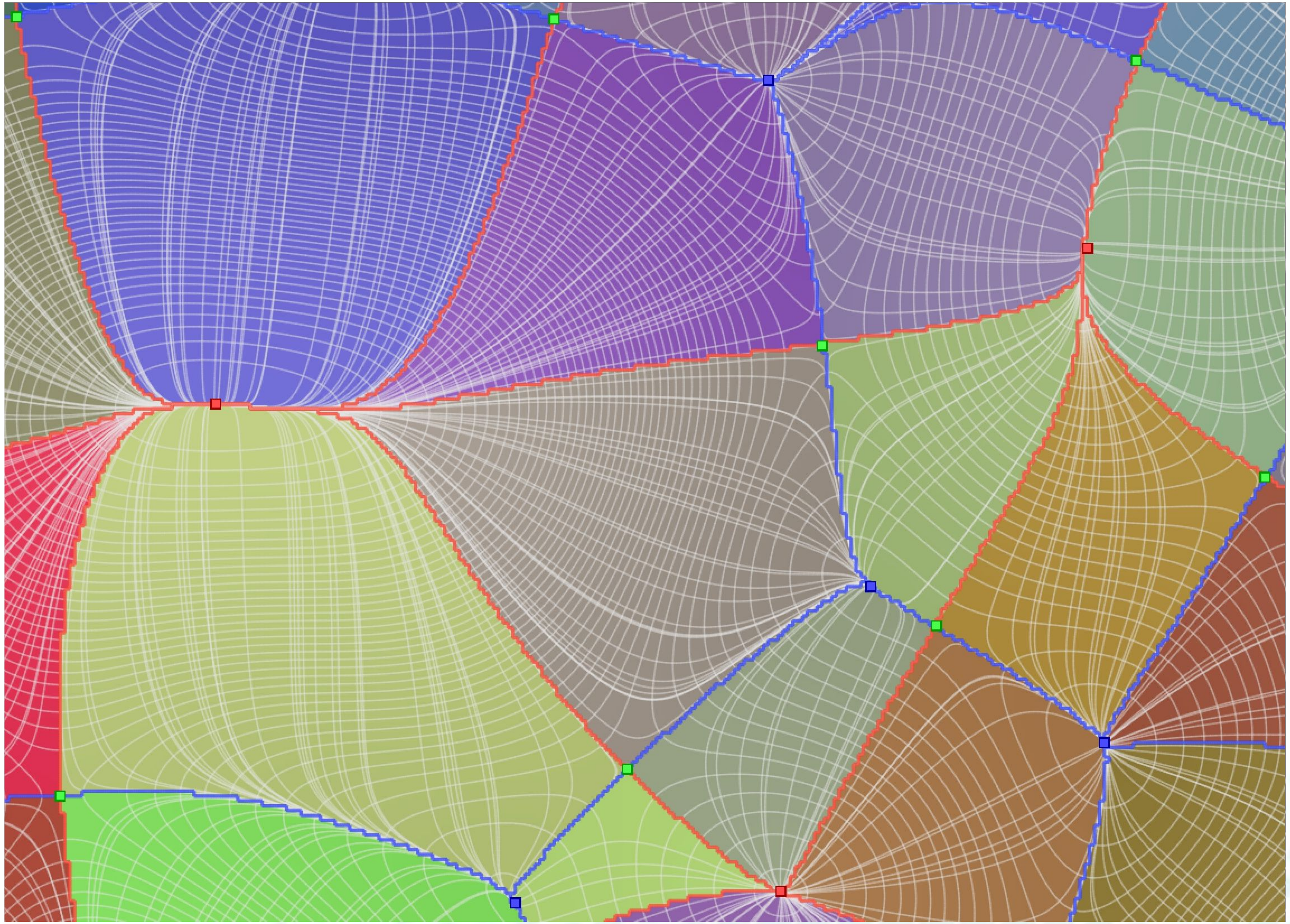
Features of a 2-dimensional function

"Monotonicity" = pieces of contours grouped by integral lines



Features of a 2-dimensional function

"Monotonicity" = pieces of contours grouped by integral lines



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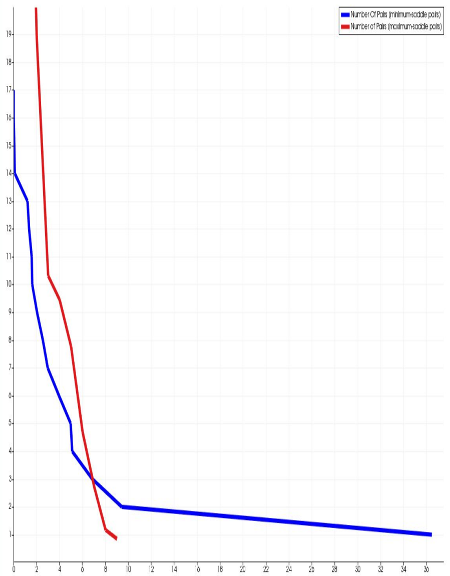
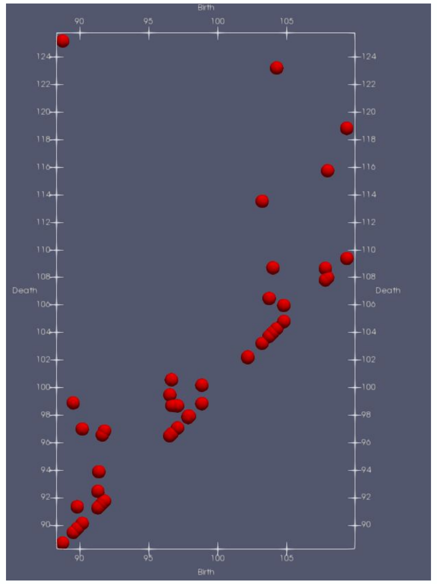
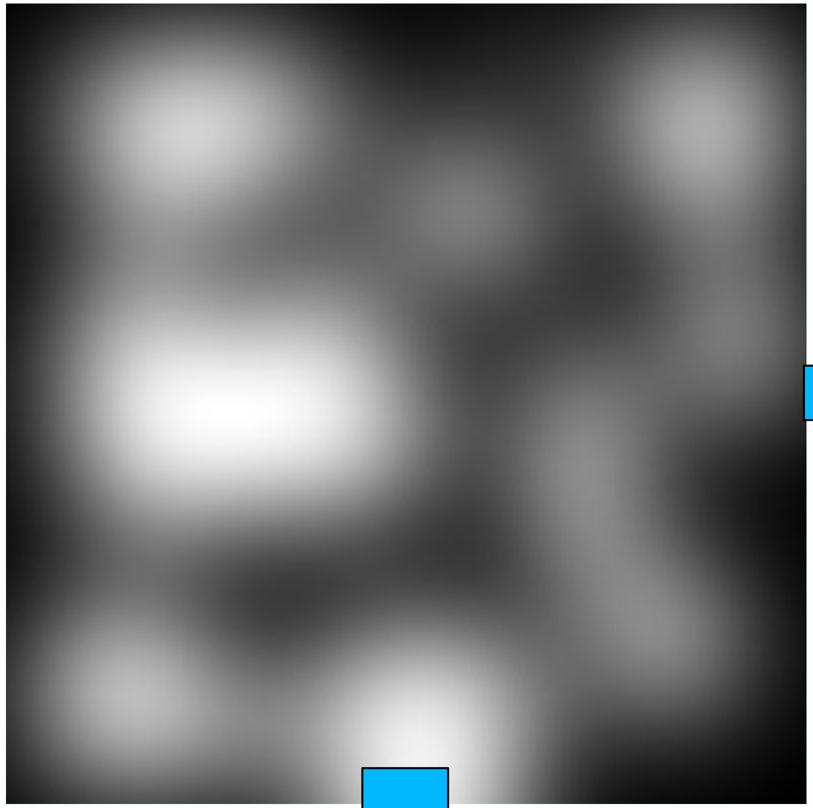
Features of a 2-dimensional function

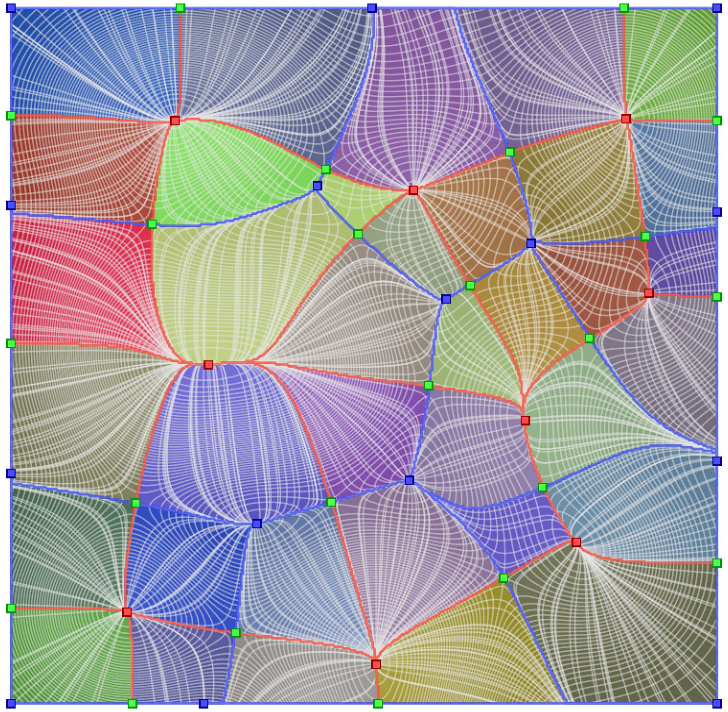


Persistence curve

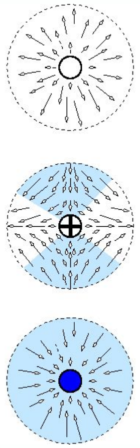


Persistence diagram





Morse-Smale Complex

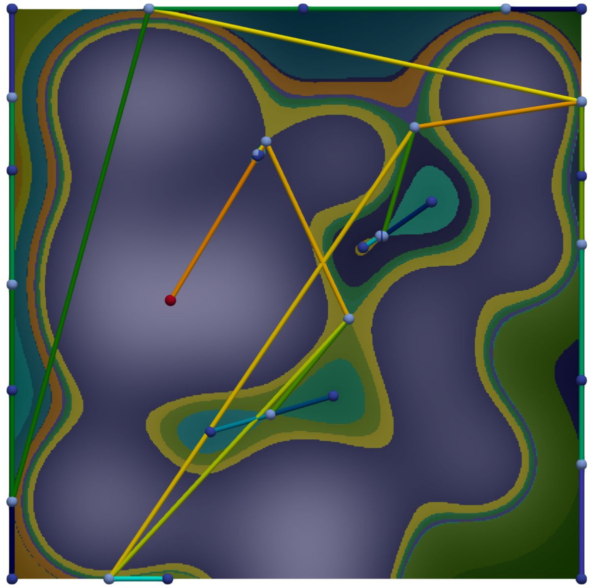


Minimum

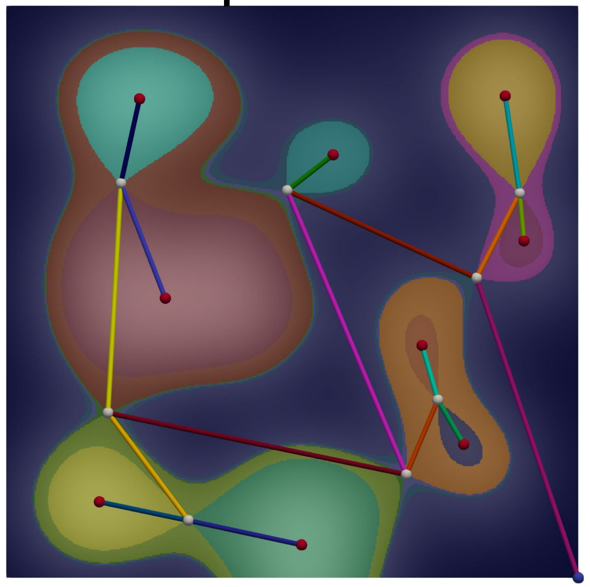
Saddle

Maximum

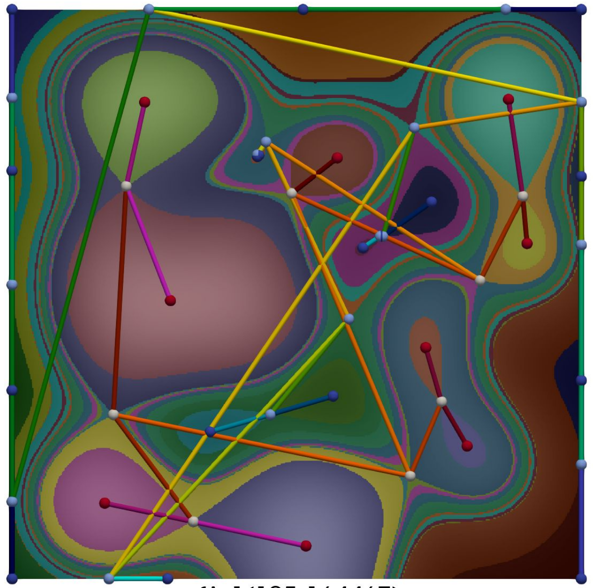
Join tree



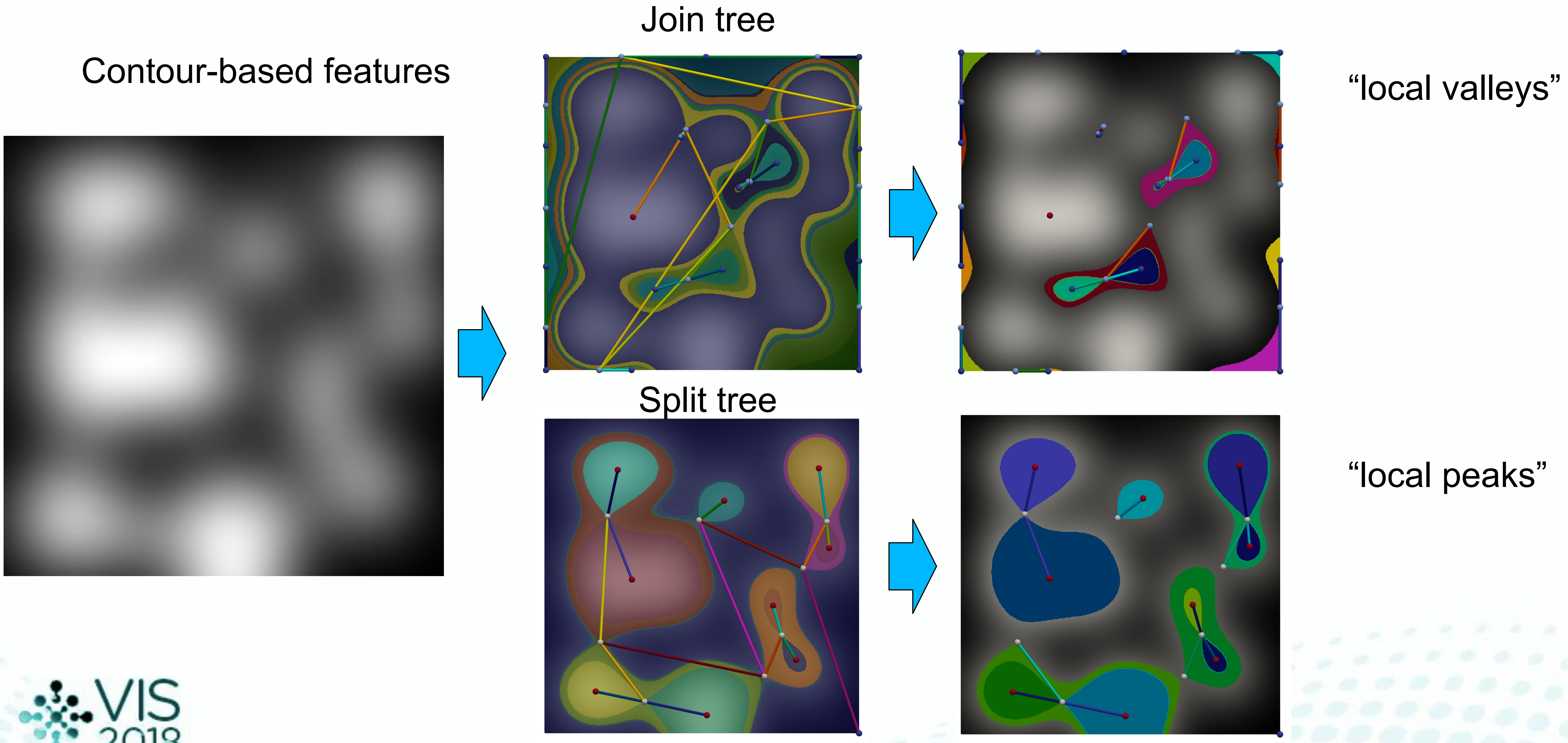
Split tree



Contour tree

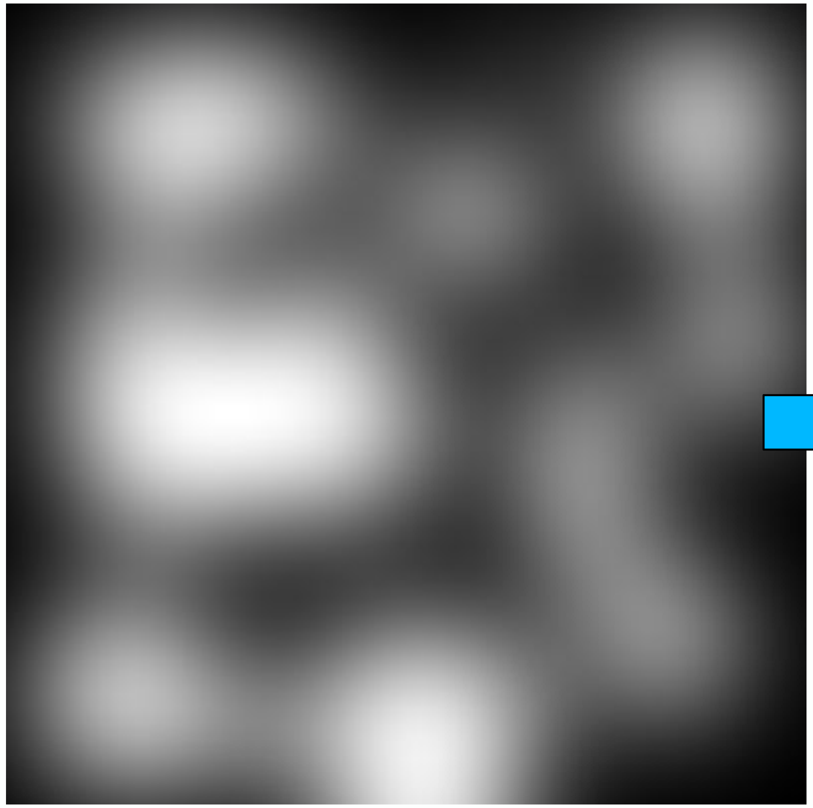


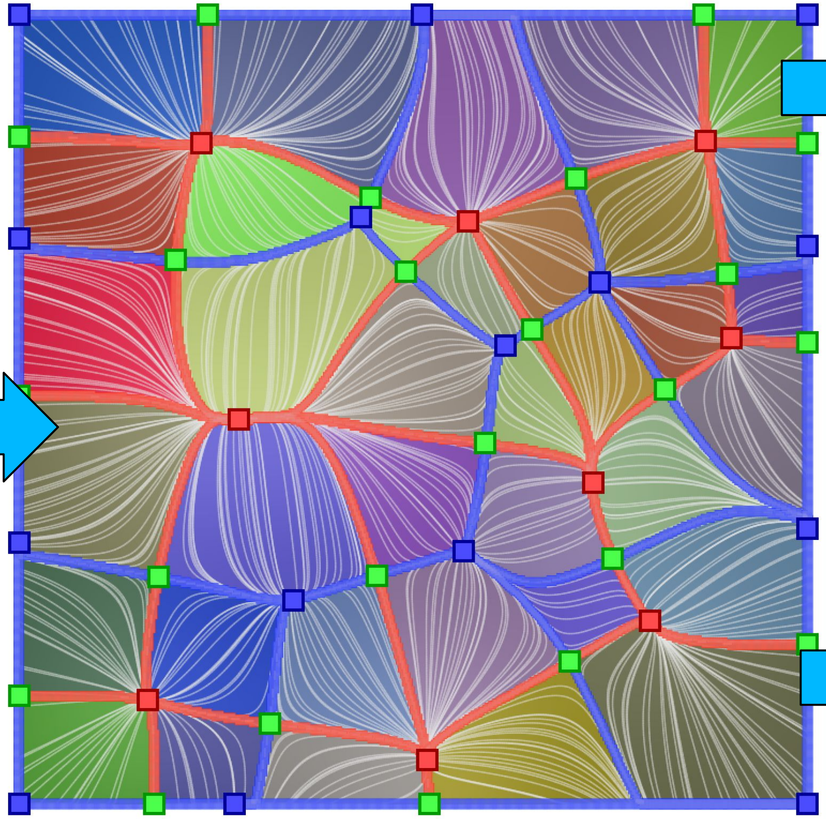
Features of a 2-dimensional function



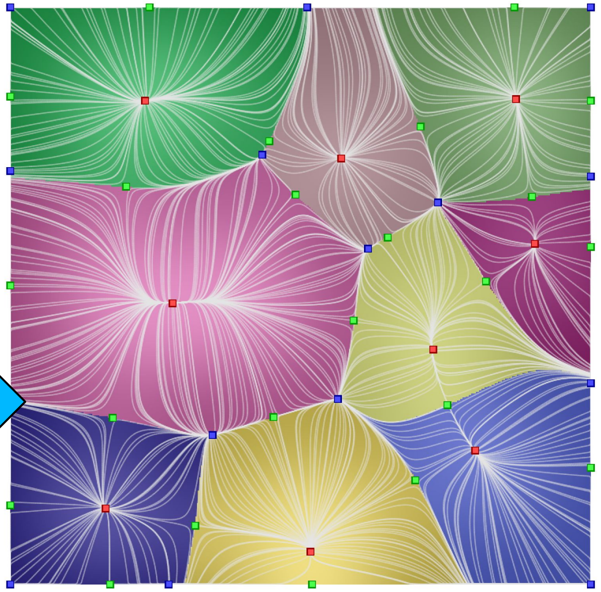
Features of a 2-dimensional function

Gradient based features

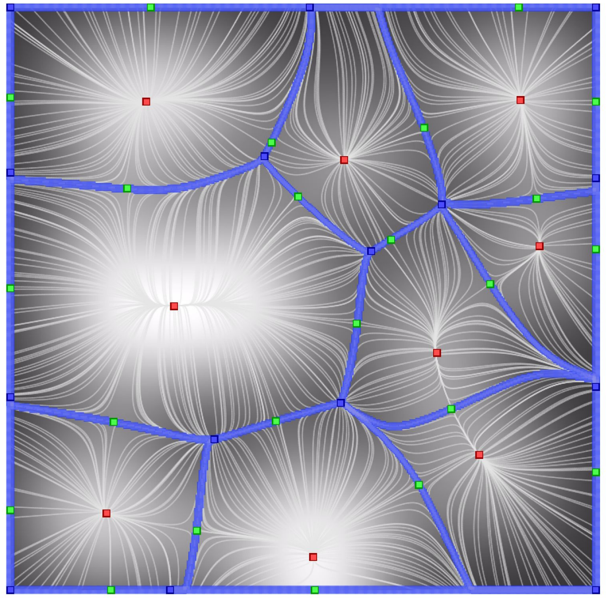




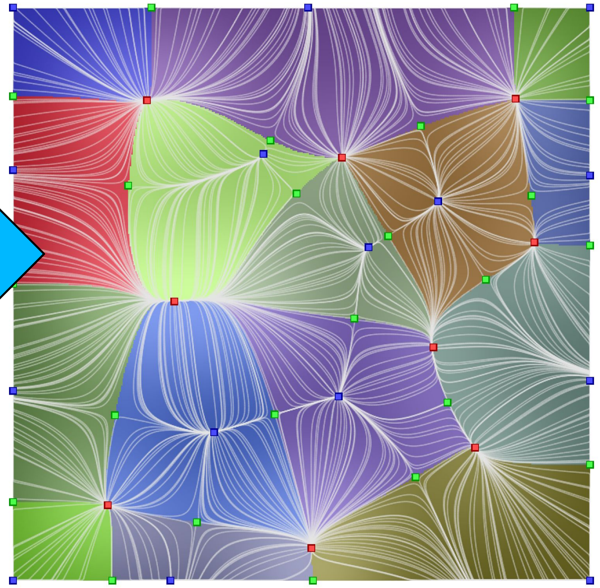
Morse-Smale Complex



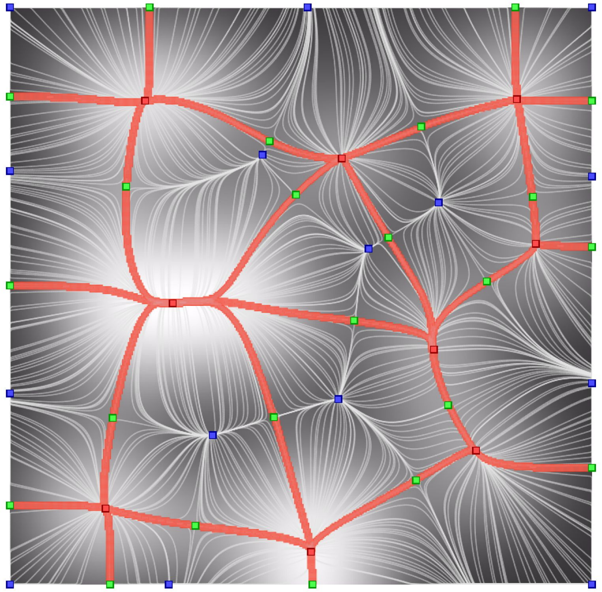
“Mountains”



“Valley lines”

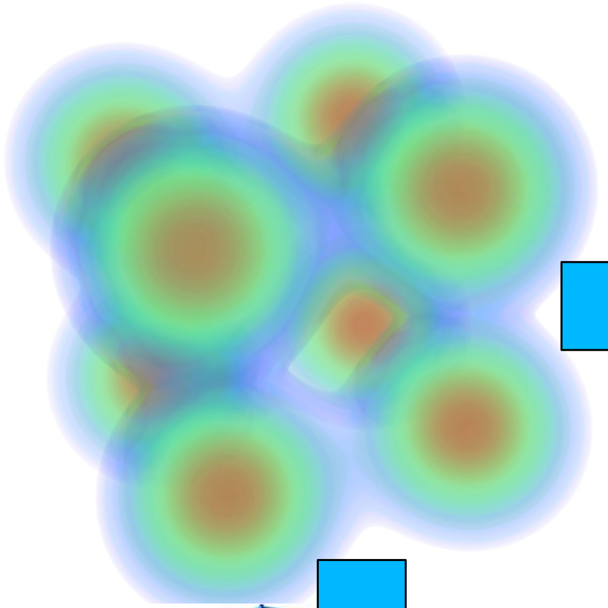


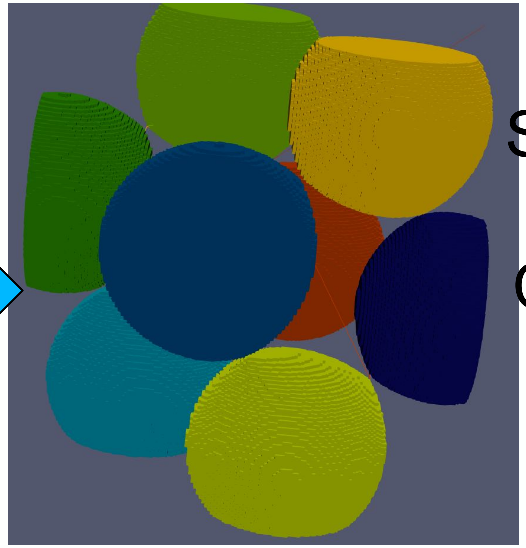
“Basins”



“Ridge lines”

Features of a 3-dimensional function





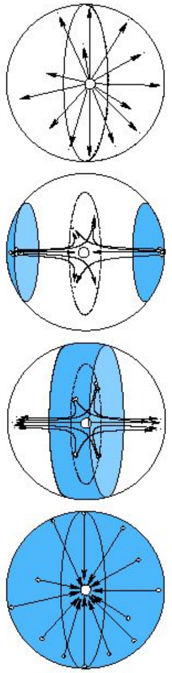
Split tree, Join tree, Contour-trees works the same as 1- and 2d

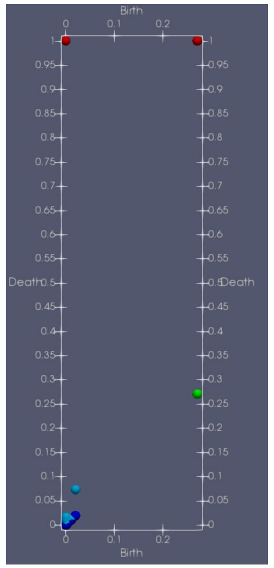
Minimum

1-saddle

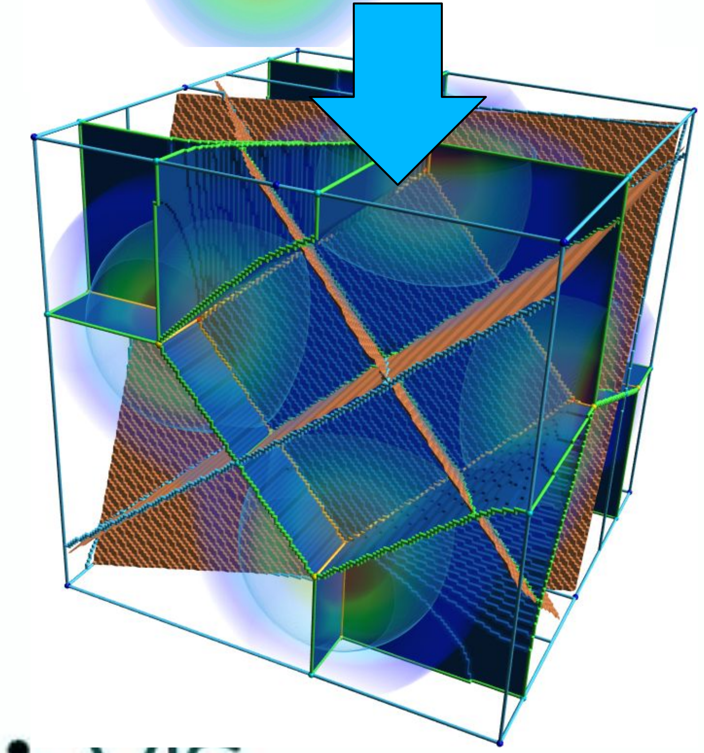
2-saddle

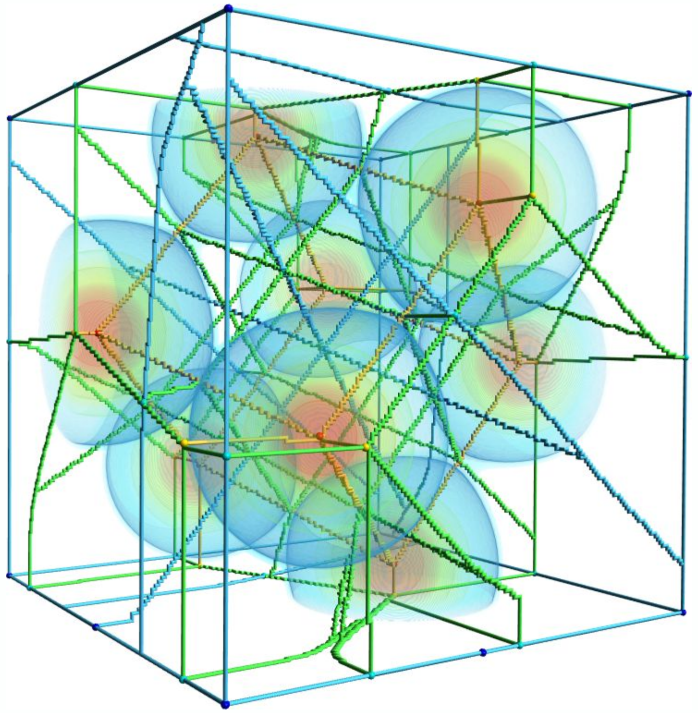
Maximum



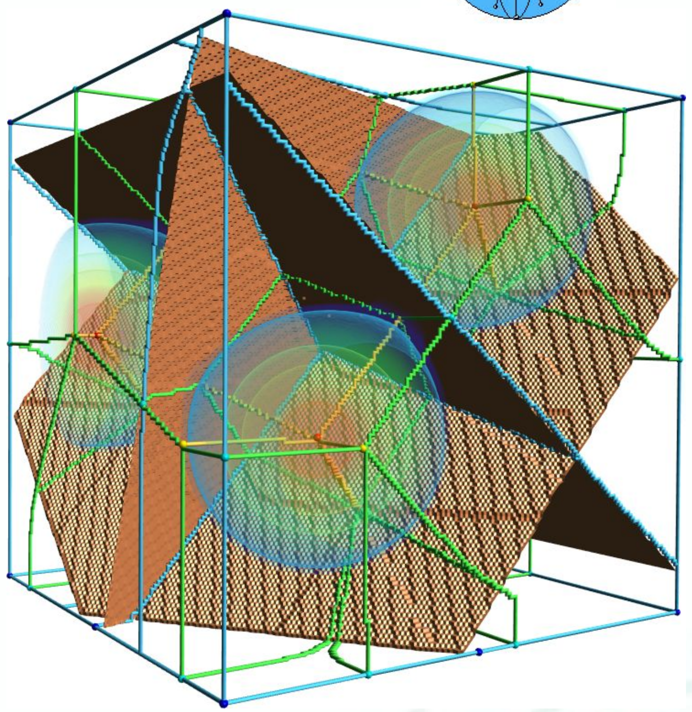


Persistence diagrams also record saddle-saddle pairs

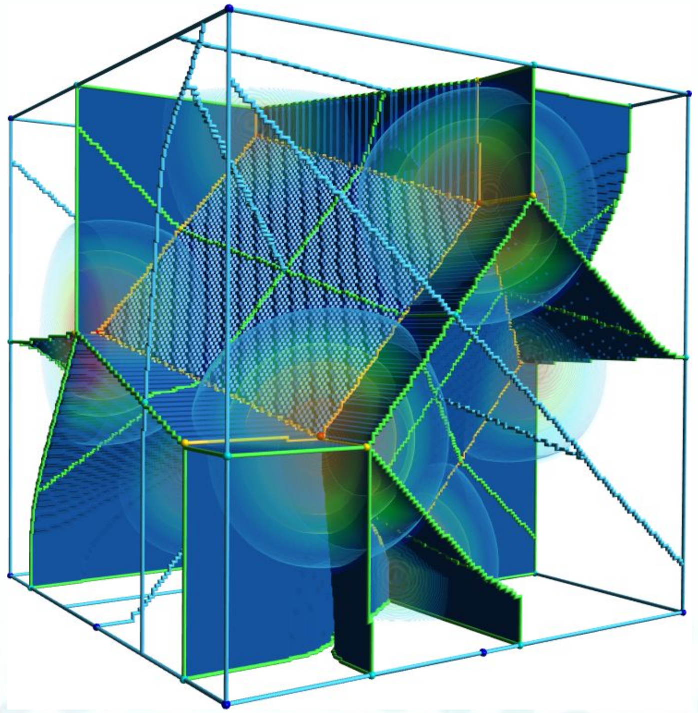




“Ridge lines” “Valley lines”
“Saddle connectors”



“Ridge surfaces”
Descending 2-manifolds



“Valley surfaces”
Ascending 2-manifolds

How do we approach an analysis problem?

In reverse:

"I need to extract/identify/count/measure…."

Extreme point	Persistence diagram/ persistence plot
Local peaks/valleys	Leaves/branches of join/split/contour trees
Ridge/Valley lines	Saddle-extremum arcs of MSC
Mountains/Basins	Ascending/descending segmentation ids of MSC
Separating surfaces	Ascending/descending 2-manifolds of MSC

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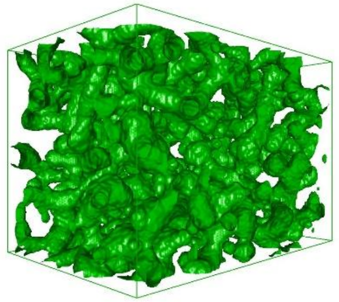
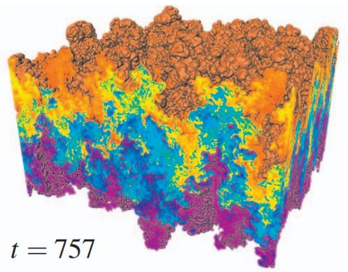
Create a hypothesis for topological features

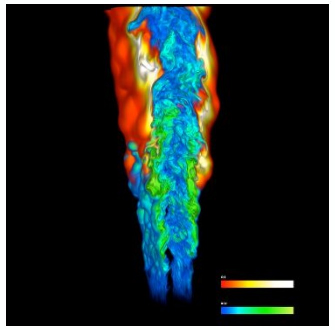
Data — Scientific question — Redefine with abstraction



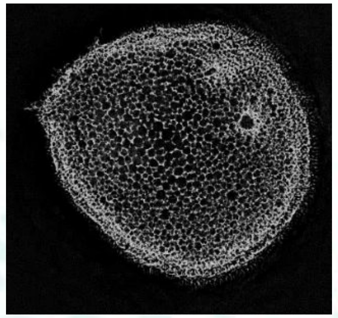
→ “Quantify porosity” → Measure total length and number of cycles in ridge-like “filaments”



→ “Measure bubble formation rate” → Evolution of stable descending 2-manifolds (“mountains”) on mixture fraction isosurface

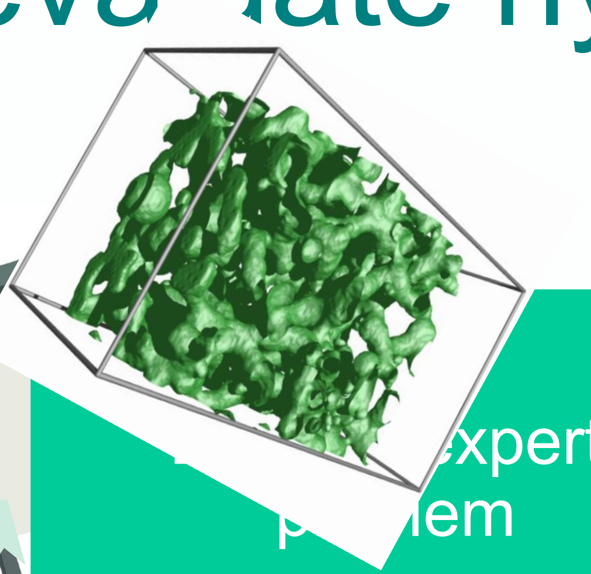


→ “Identify ignition kernels” → Local peaks and regions in temperature field below flame base



→ “Measure deformation of cavity walls” → Measure deviation from plane for ascending 2-manifolds separating “basins”

Continuously re-evaluate hypothesis



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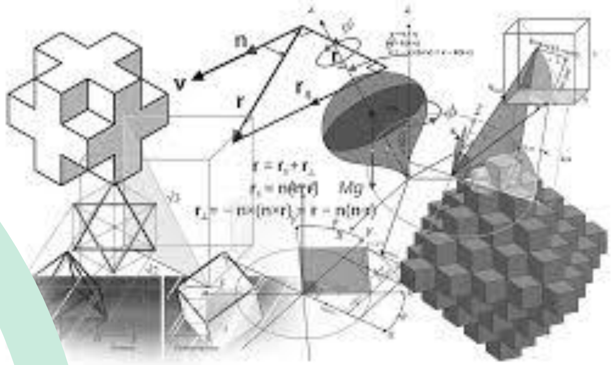
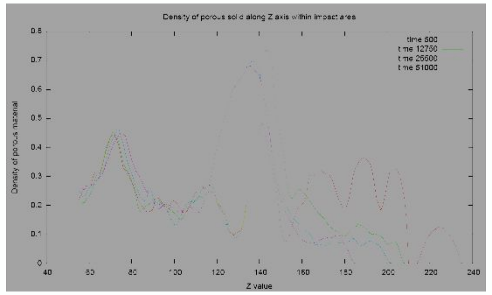
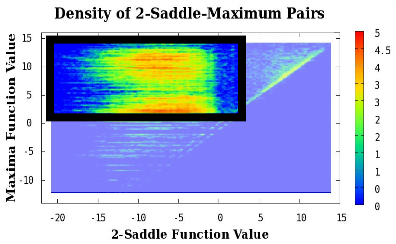
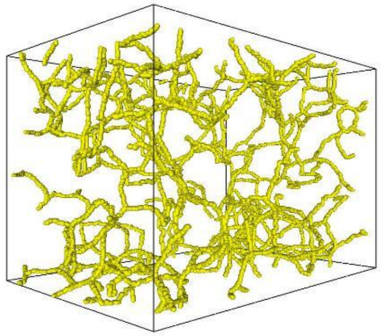




Visualization/Analysis

Proposed abstraction



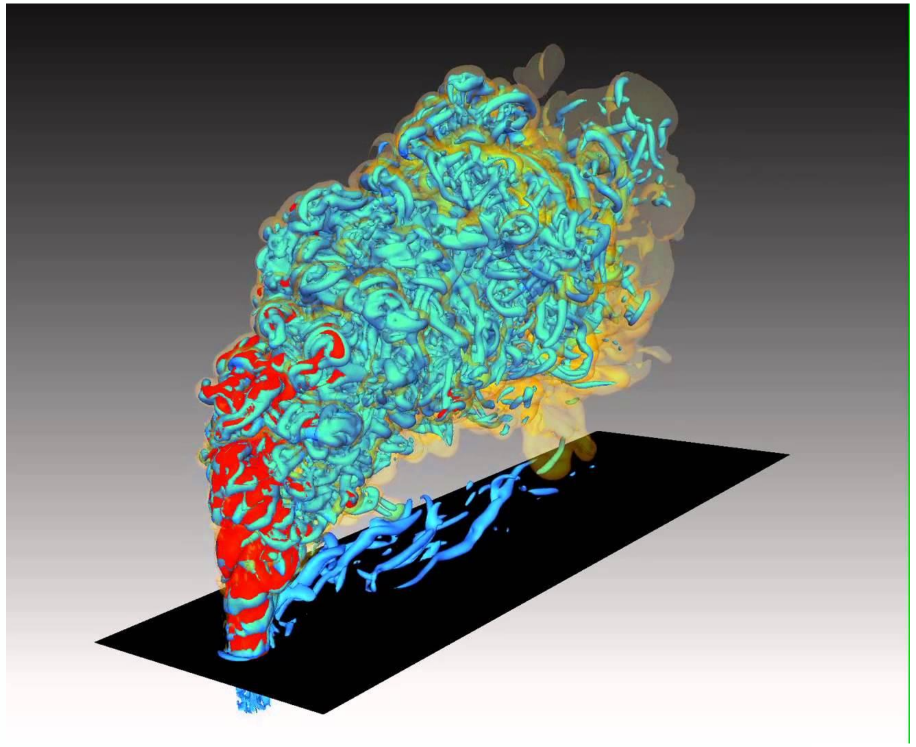


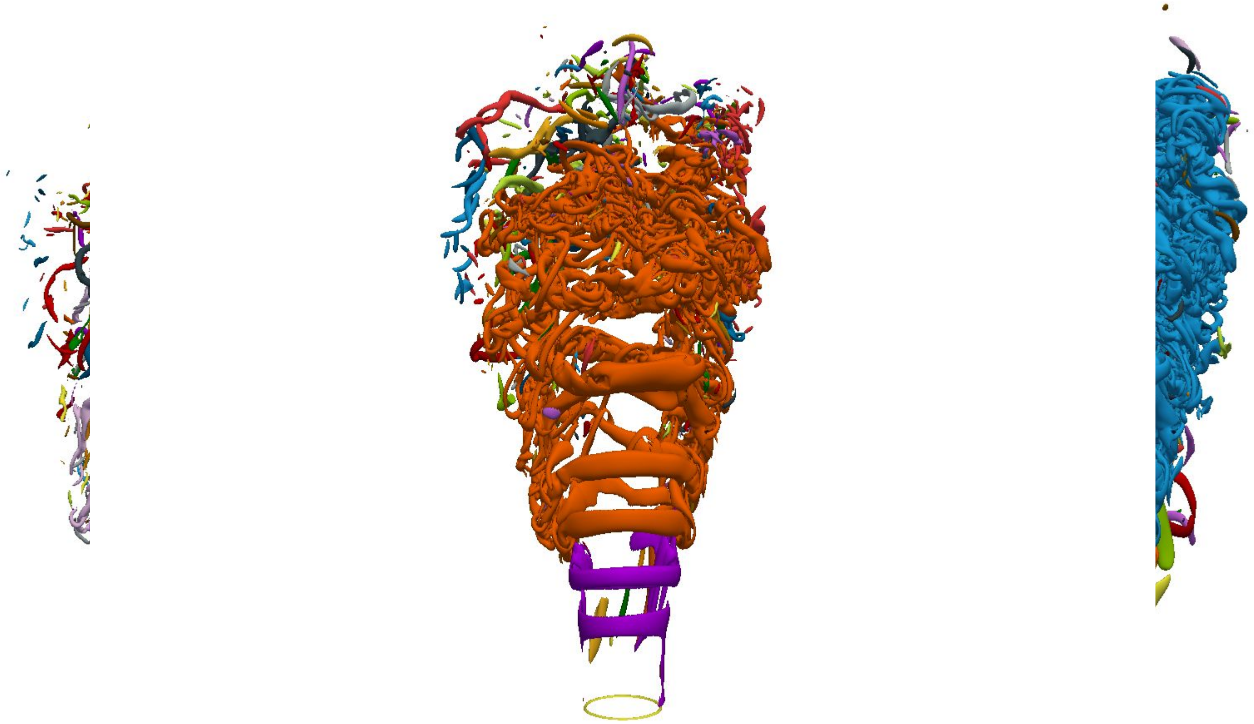
Computational realization

Some examples

Identifying vorticity structures in turbulent flows

Indicator-based Vortex Detection Remains most Common Approach with Well-Known Problems





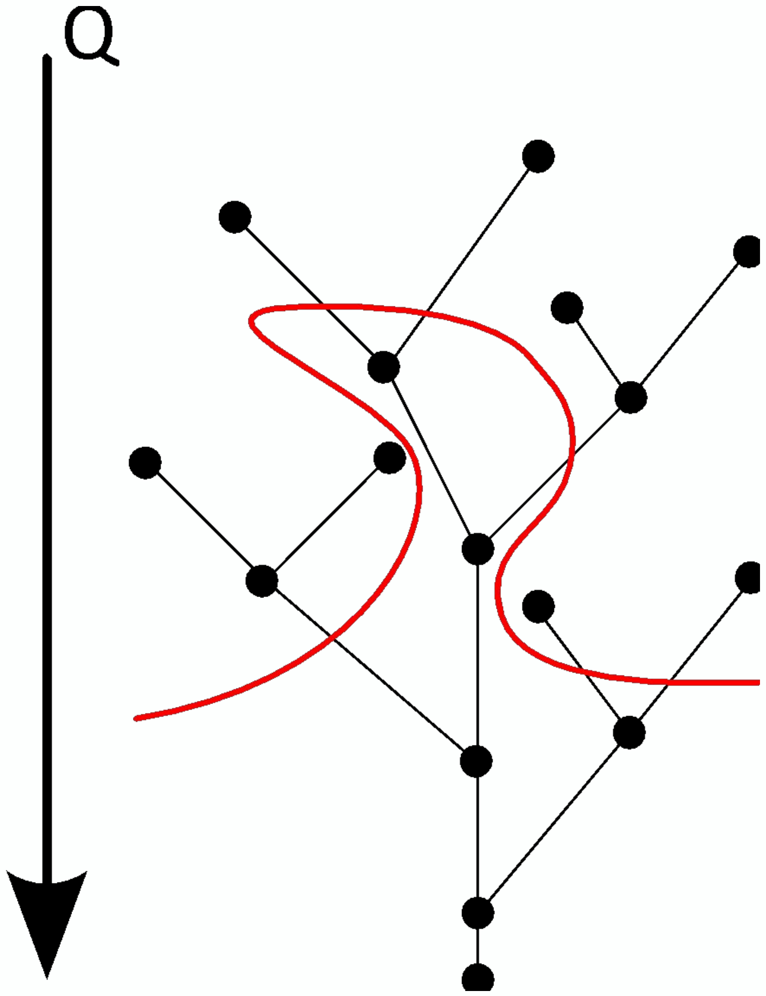
Q = 1000　　Q = 100　　Q = 5

P.-T. Bremer, A. Gruber, J. Bennett, A. Gyulassy, H. Kolla, J. Chen, R.W. Grout. **“Identifying turbulent structures through topological segmentation,”** In *Com. in App. Math. and Comp. Sci.*, Vol. 11, No. 1, pp. 37-53. 2016.

Identifying vorticity structures in turbulent flows

Cutting the split tree based on *relevance* allows for “local peaks” definition of vortex

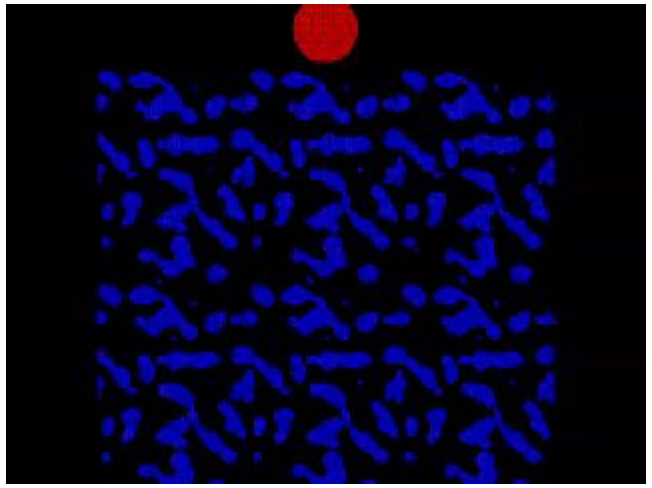
$$relevance(p) = 1 - \frac{localMax(p) - f(p)}{localMax(p) - globalMin}$$

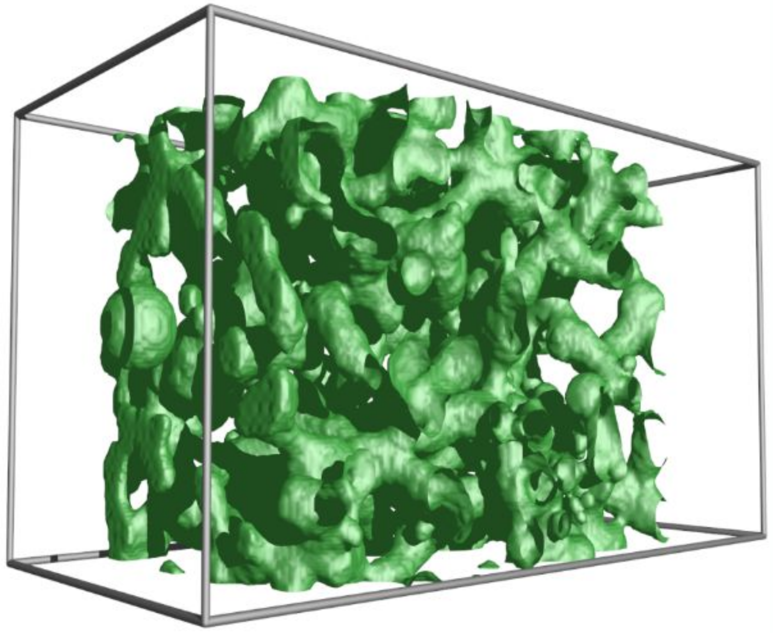


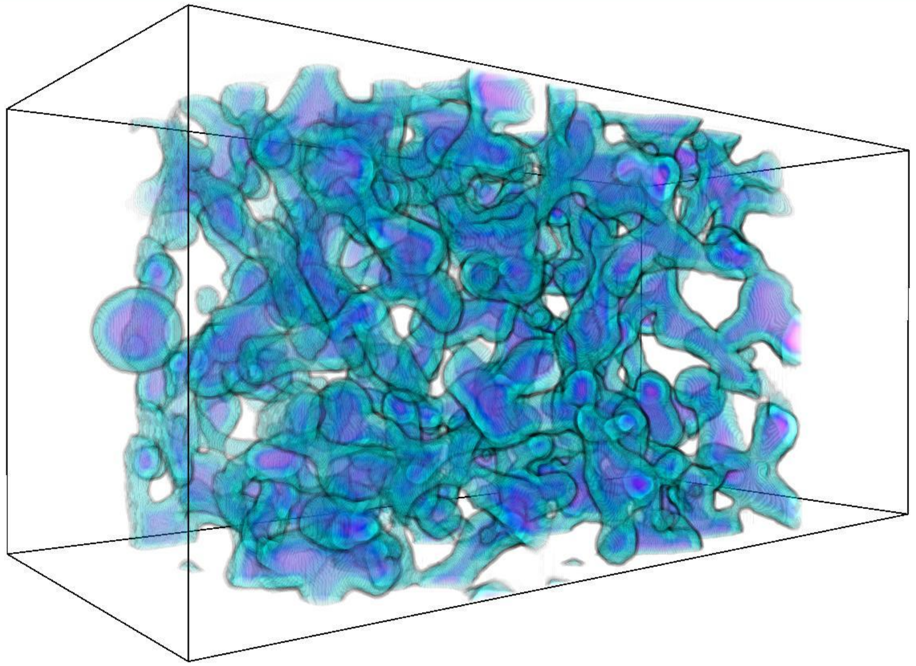
P.-T. Bremer, A. Gruber, J. Bennett, A. Gyulassy, H. Kolla, J. Chen, R.W. Grout. **“Identifying turbulent structures through topological segmentation,”** In *Com. in App. Math. and Comp. Sci.*, Vol. 11, No. 1, pp. 37-53. 2016.

Measure change in porosity of metal foam

Measure filament length and number of cycles, using ridge-like lines of the MS complex







Time varying field of binary data (1 = metal, 0 = vacuum)

Signed distance field from 0.5 isosurface

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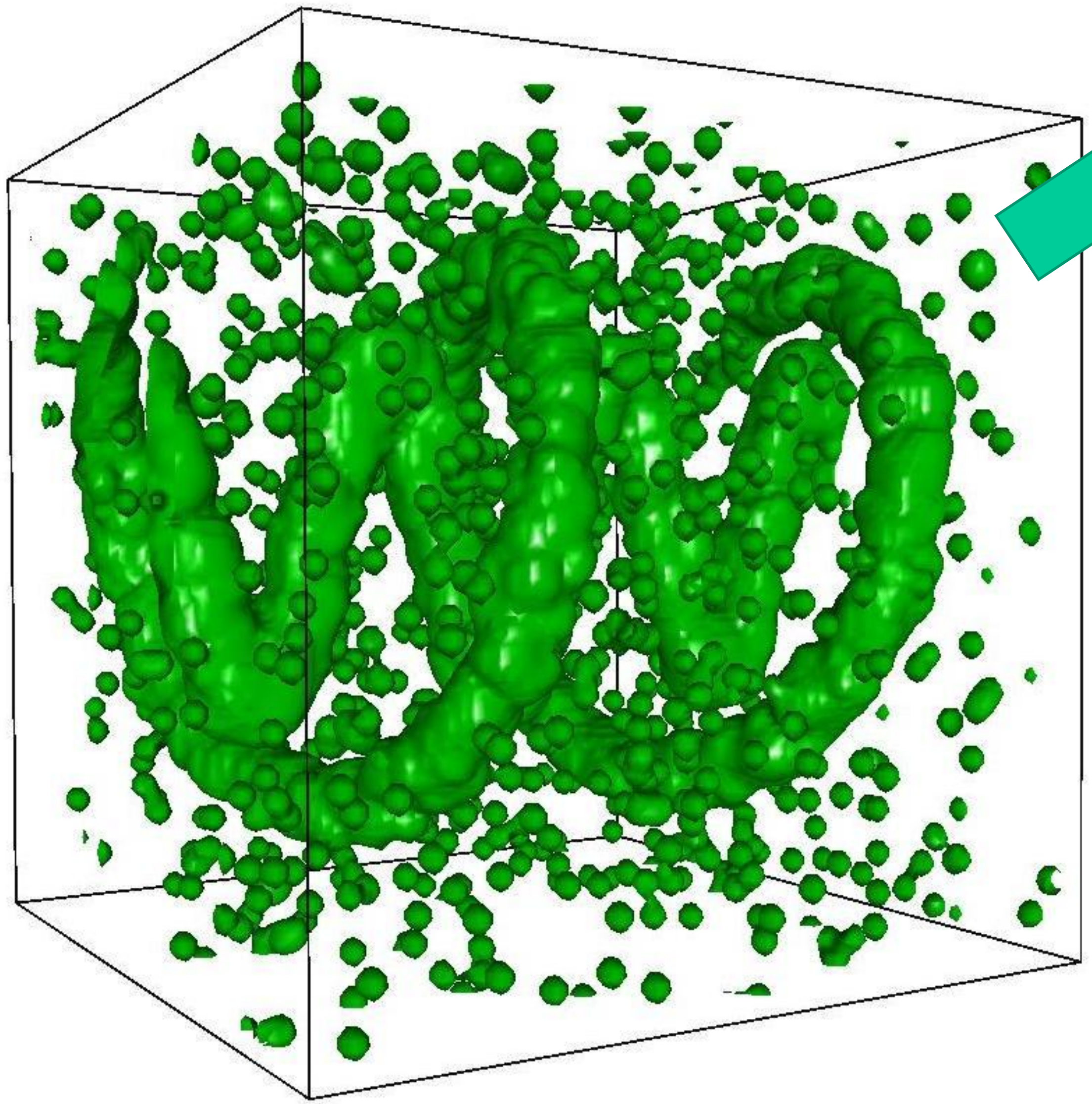
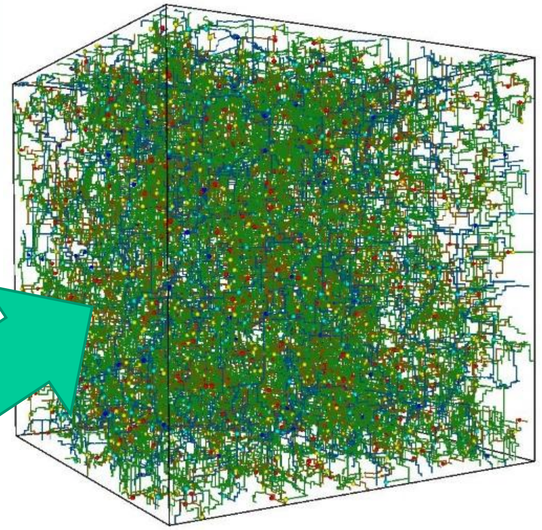
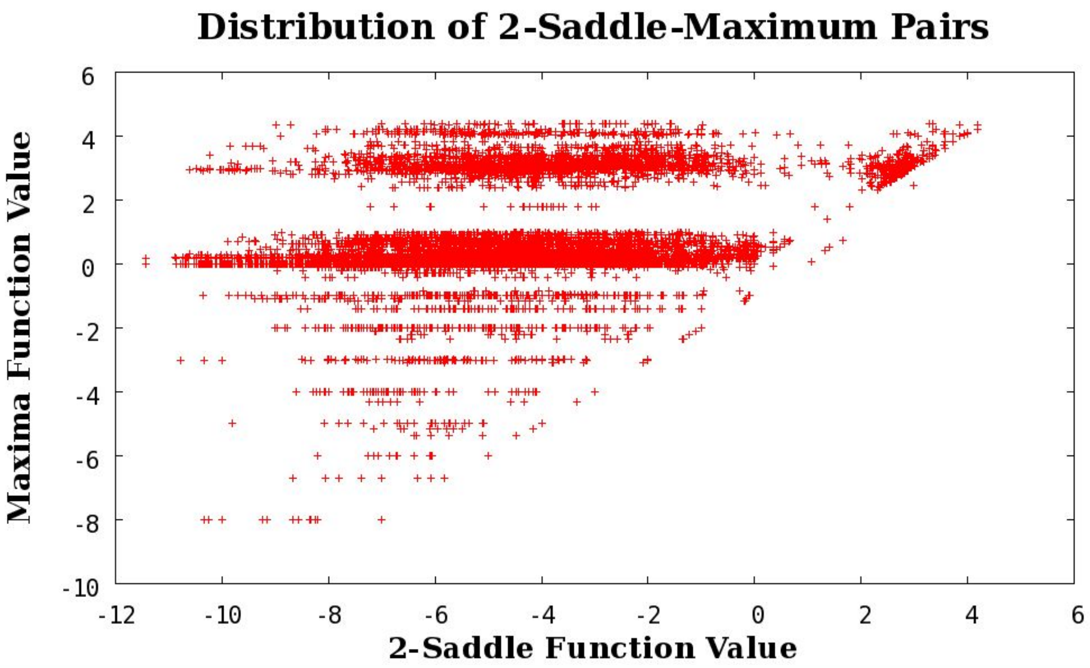
A. Gyulassy et al., **"Topologically Clean Distance Fields,"** in IEEE Transactions on Visualization and Computer Graphics, vol. 13, no. 6, pp. 1432-1439, Nov.-Dec. 2007.

Measure change in porosity of metal foam

Sanity checking our abstraction!



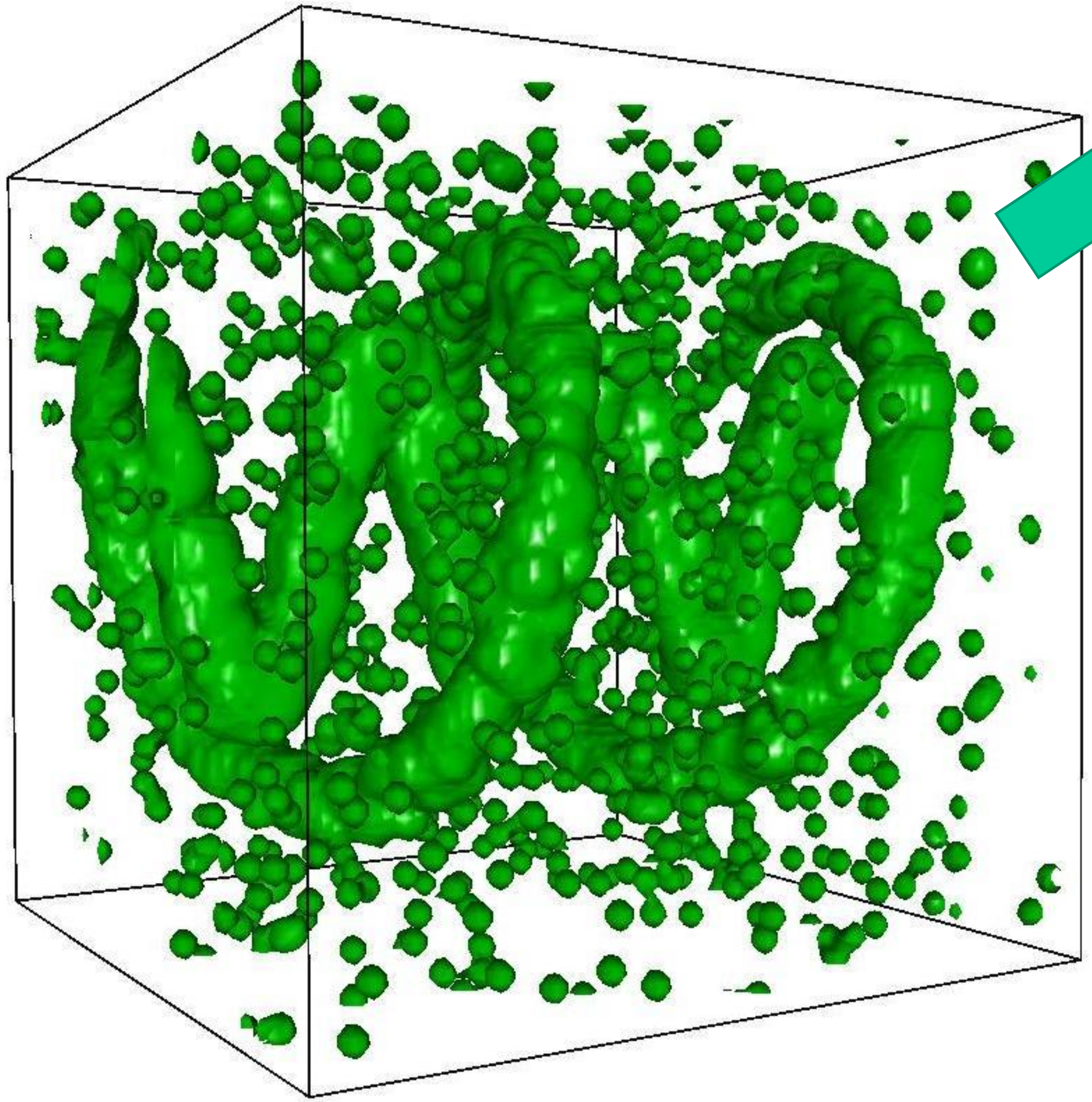


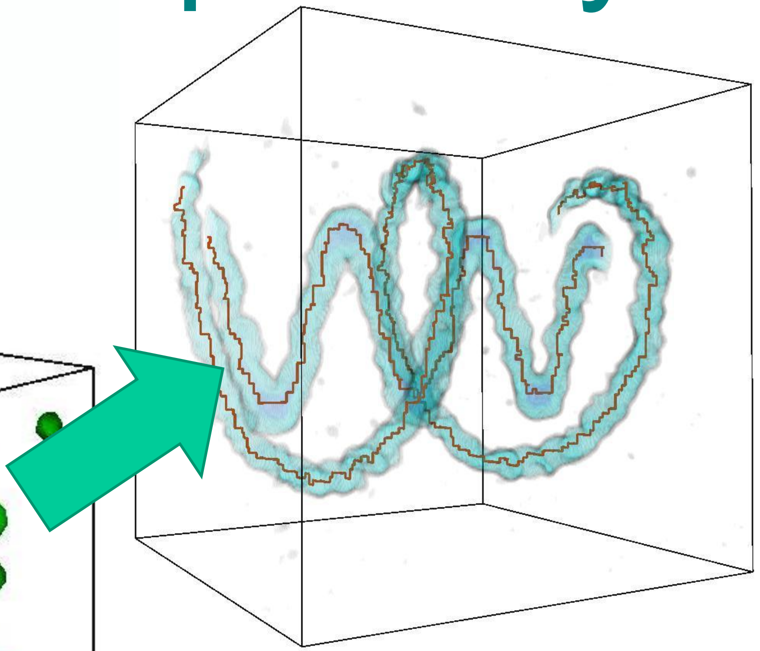


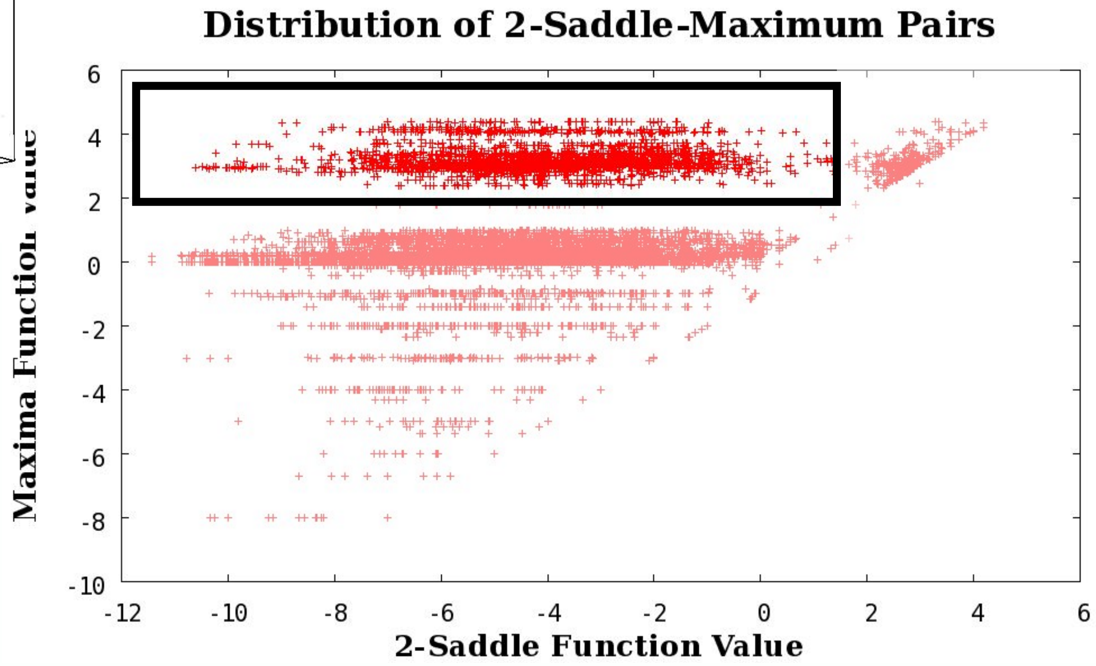
2018

Measure change in porosity of metal foam

Sanity checking our abstraction!

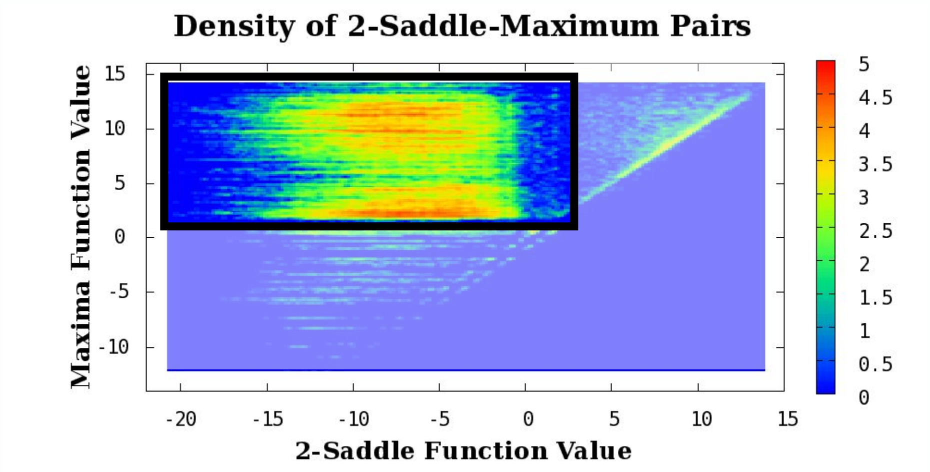


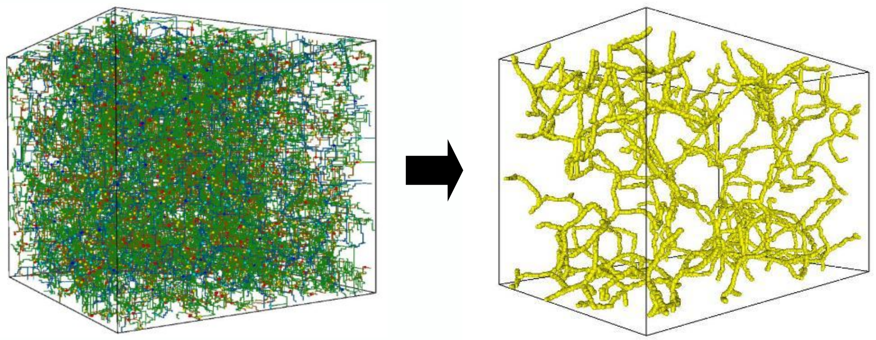


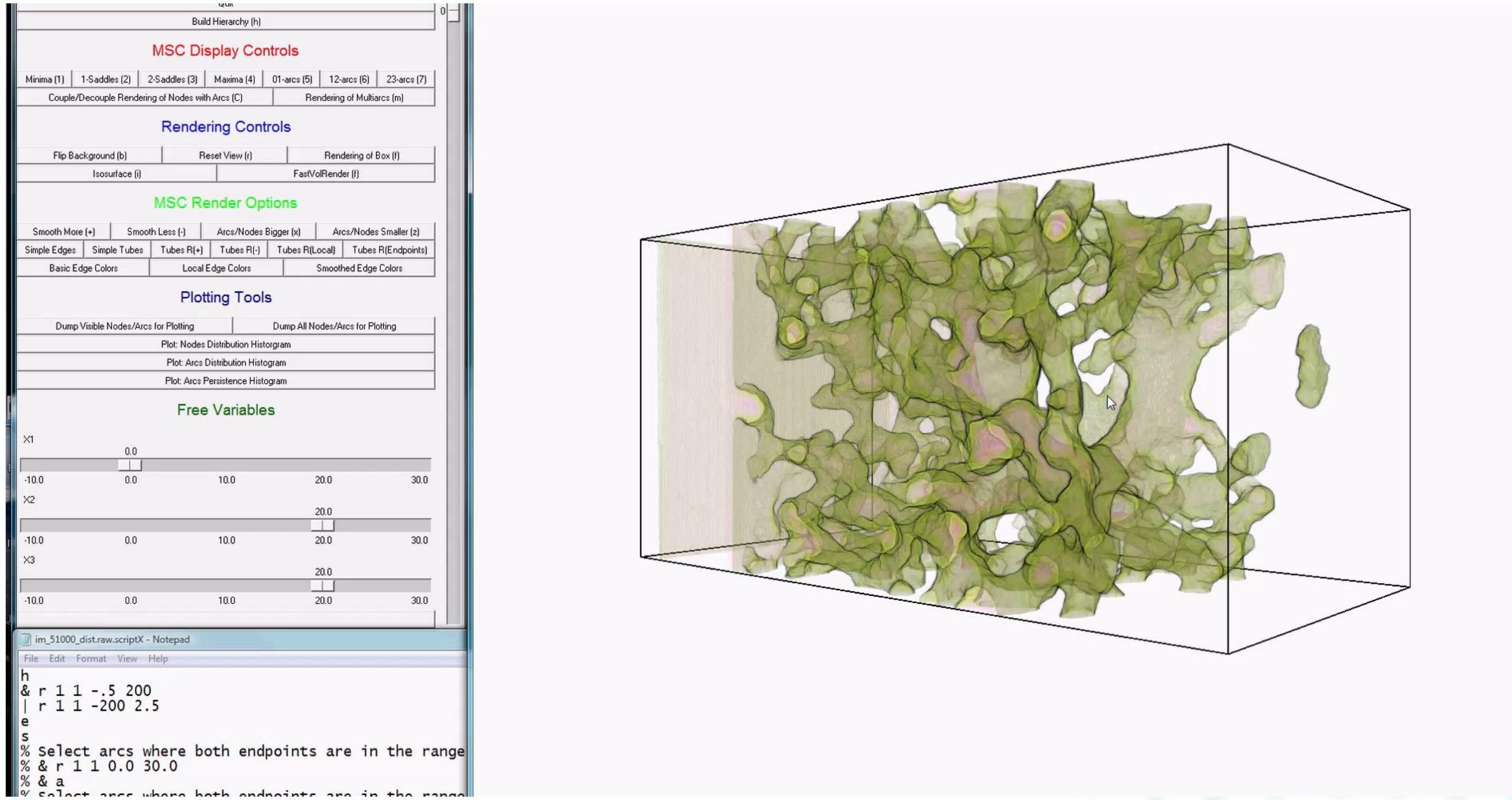


Measure change in porosity of metal foam

Measure filament length and number of cycles, using ridge-like lines of the MS complex

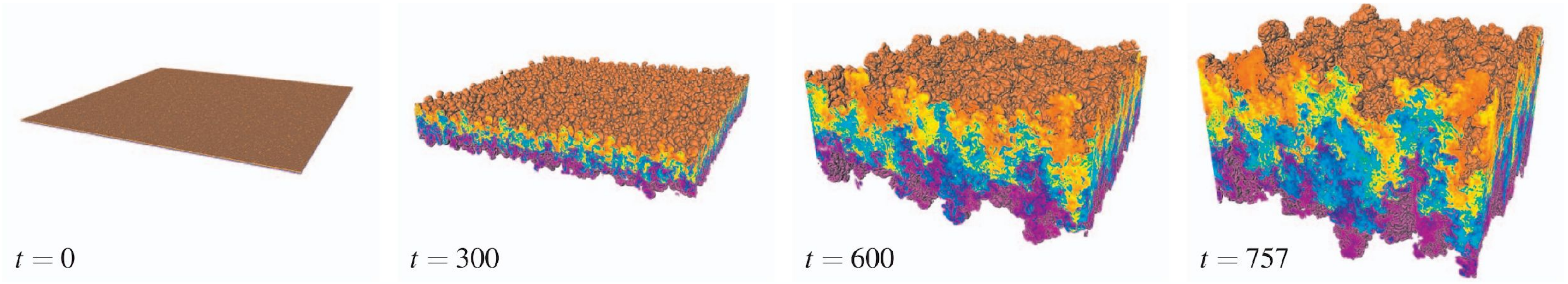






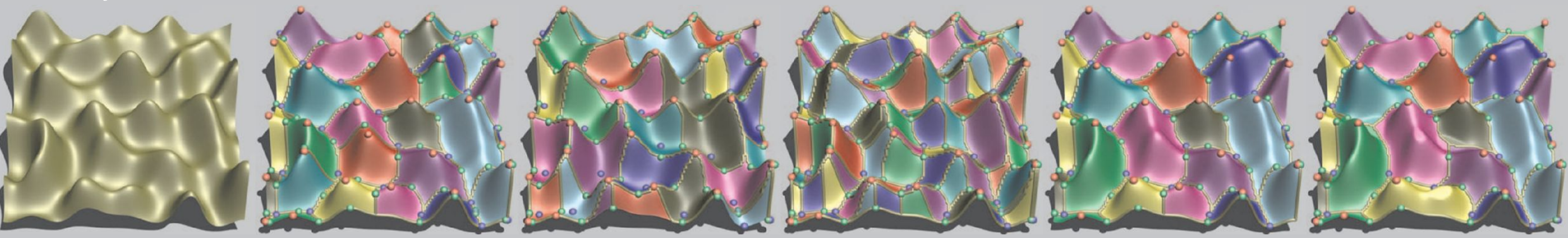
A. Gyulassy et al., **"Topologically Clean Distance Fields,"** in IEEE Transactions on Visualization and Computer Graphics, vol. 13, no. 6, pp. 1432-1439, Nov.-Dec. 2007.

Counting bubbles in turbulent mixing

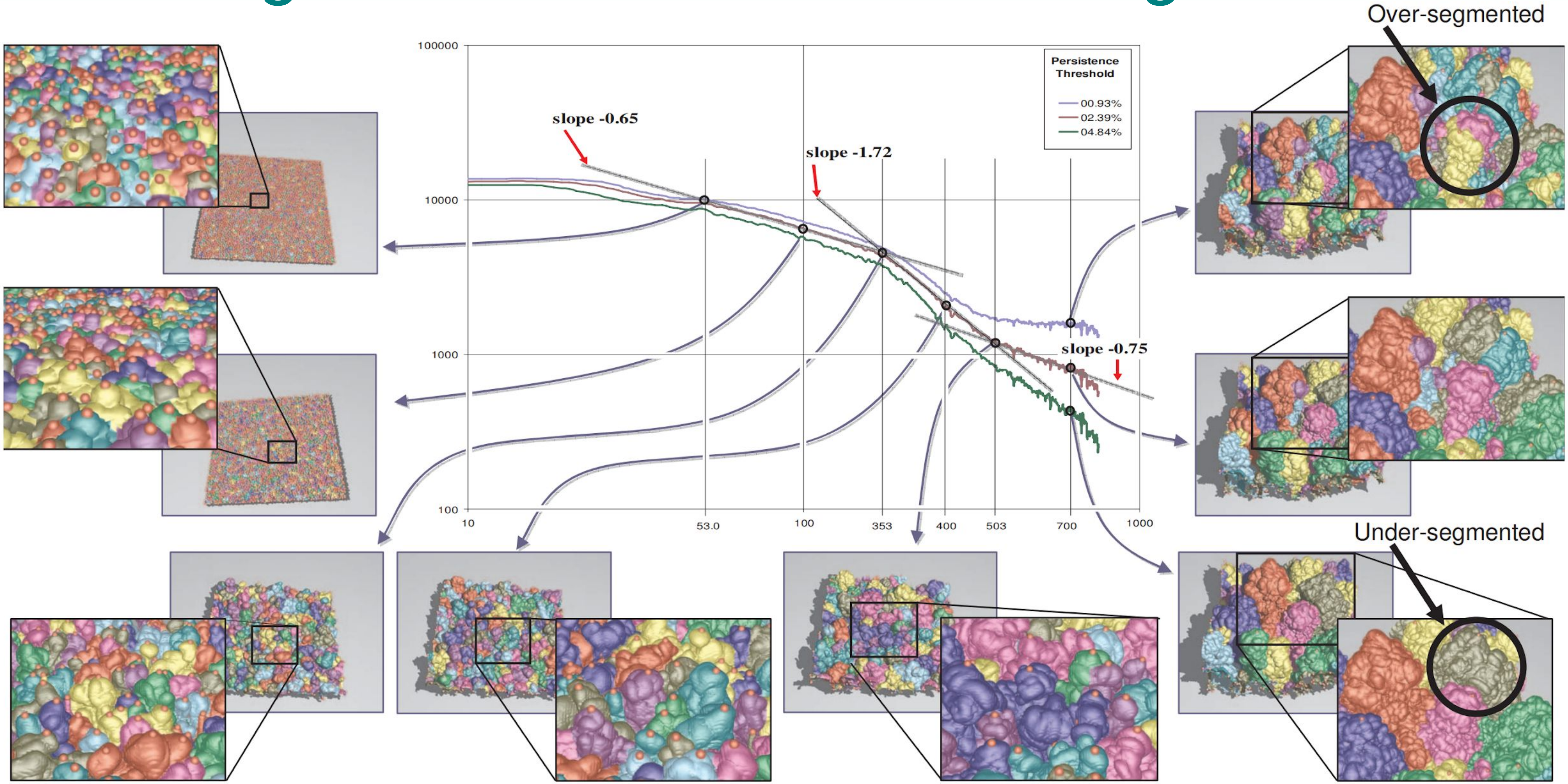


Pick a value, extract isosurface, analyze height function of it

f Descending Ascending MS-complex Simplified complex Simplified f

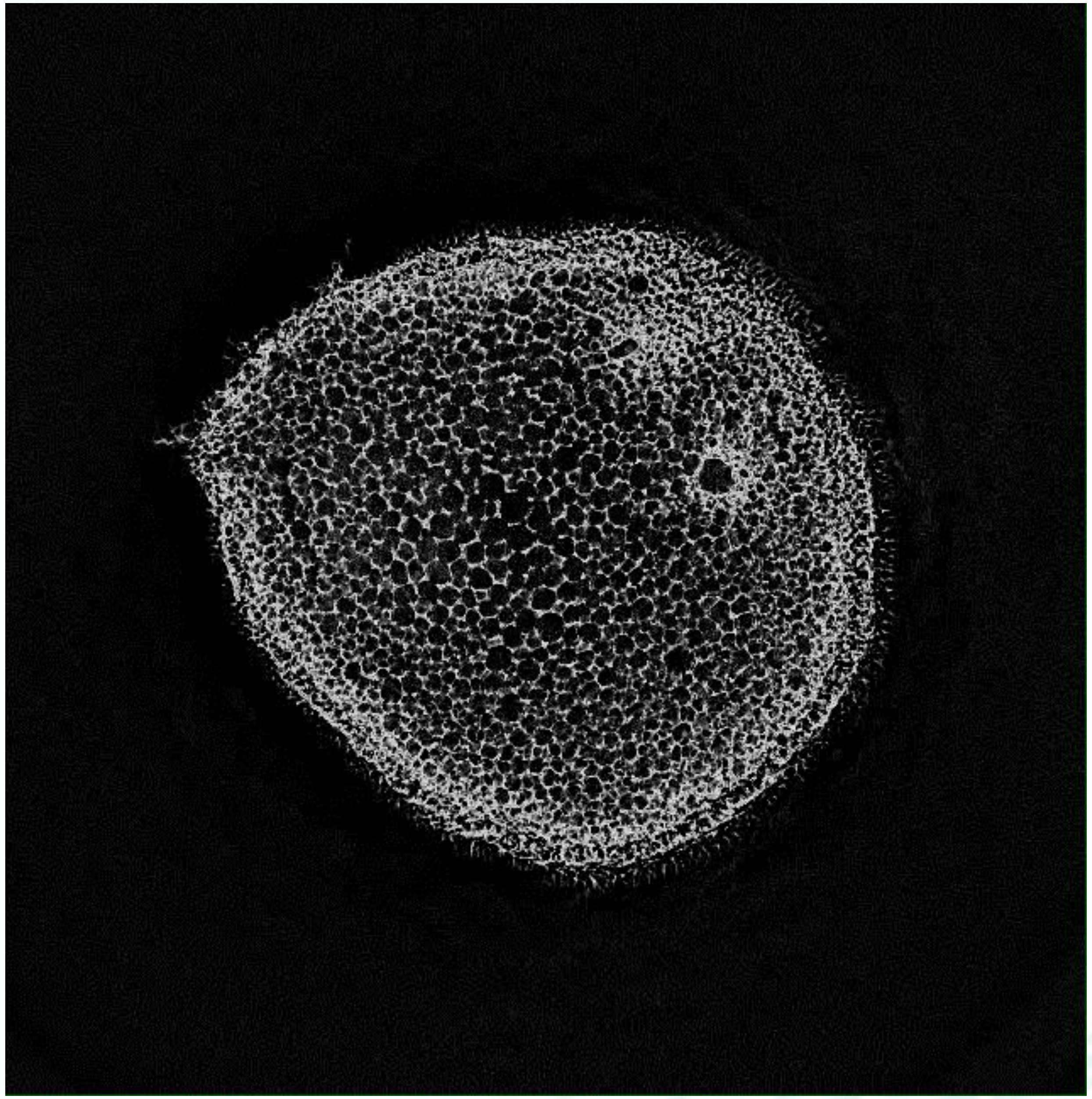


Counting bubbles in turbulent mixing



Deformation of voids in a foam

Extract basins from MS complex, measure sizes, boundaries, etc.?



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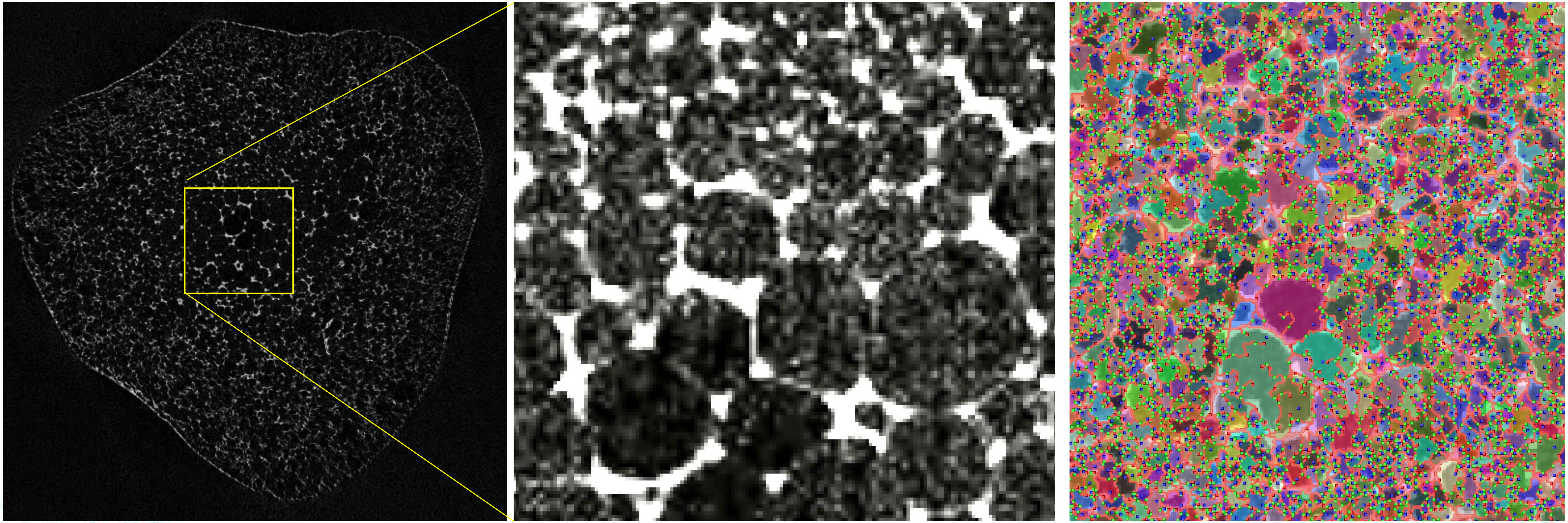

CT scans typically have high-frequency high-amplitude noise

Deformation of voids in a foam

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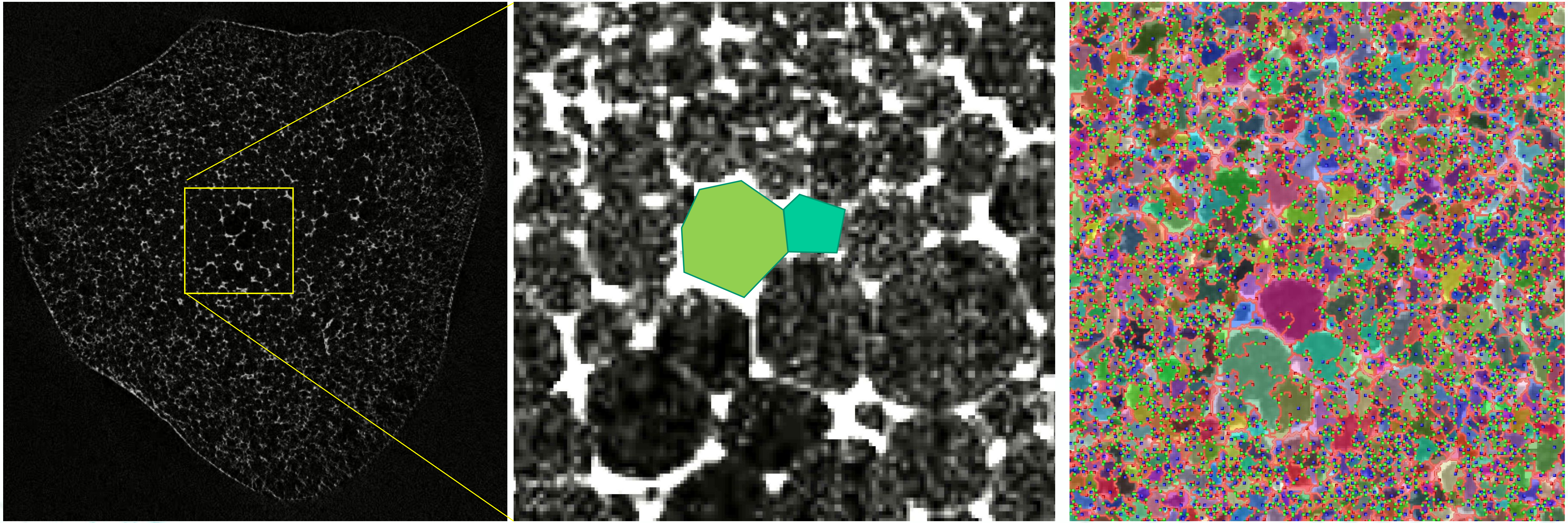

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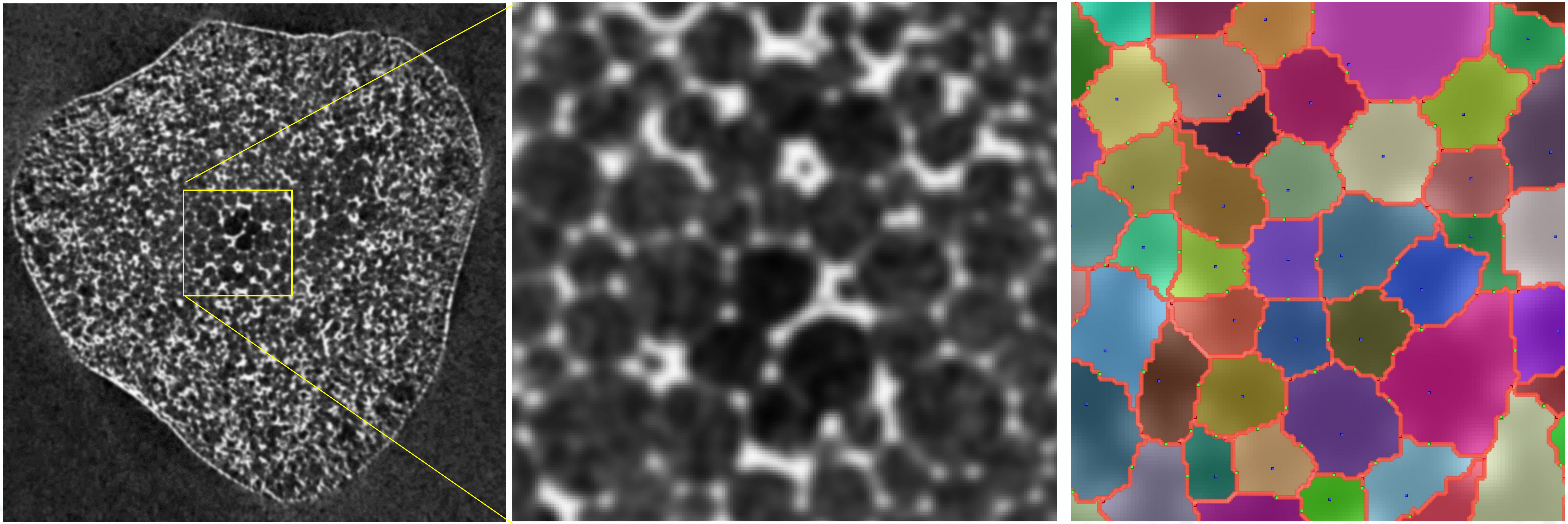


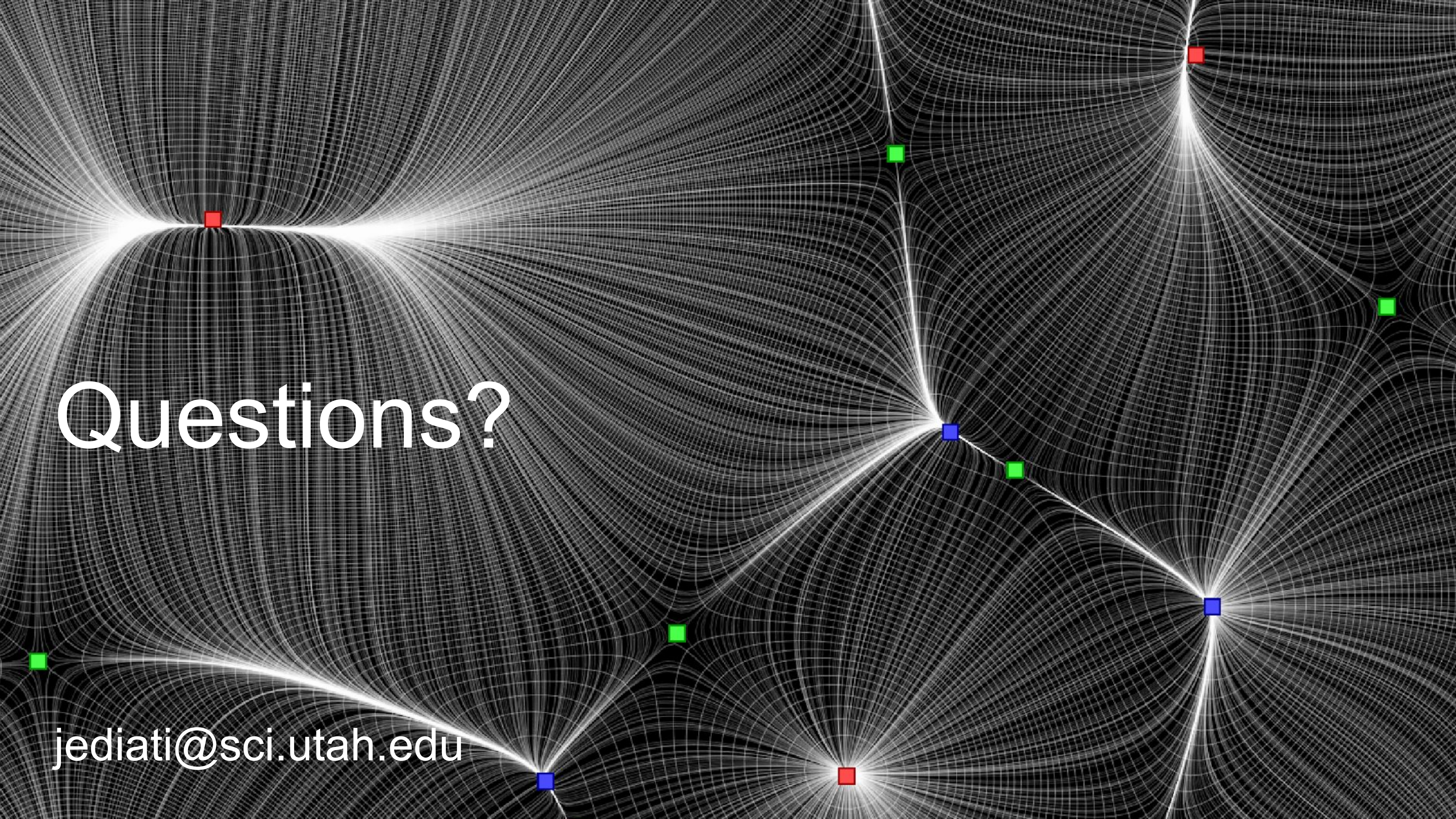
Having a model for noise, and an idea of scales can help

Words of wisdom? More like, *musings*…

I have never gotten the translation to abstraction exactly right on the first attempt (sample size 20+ application domains in 10 years)

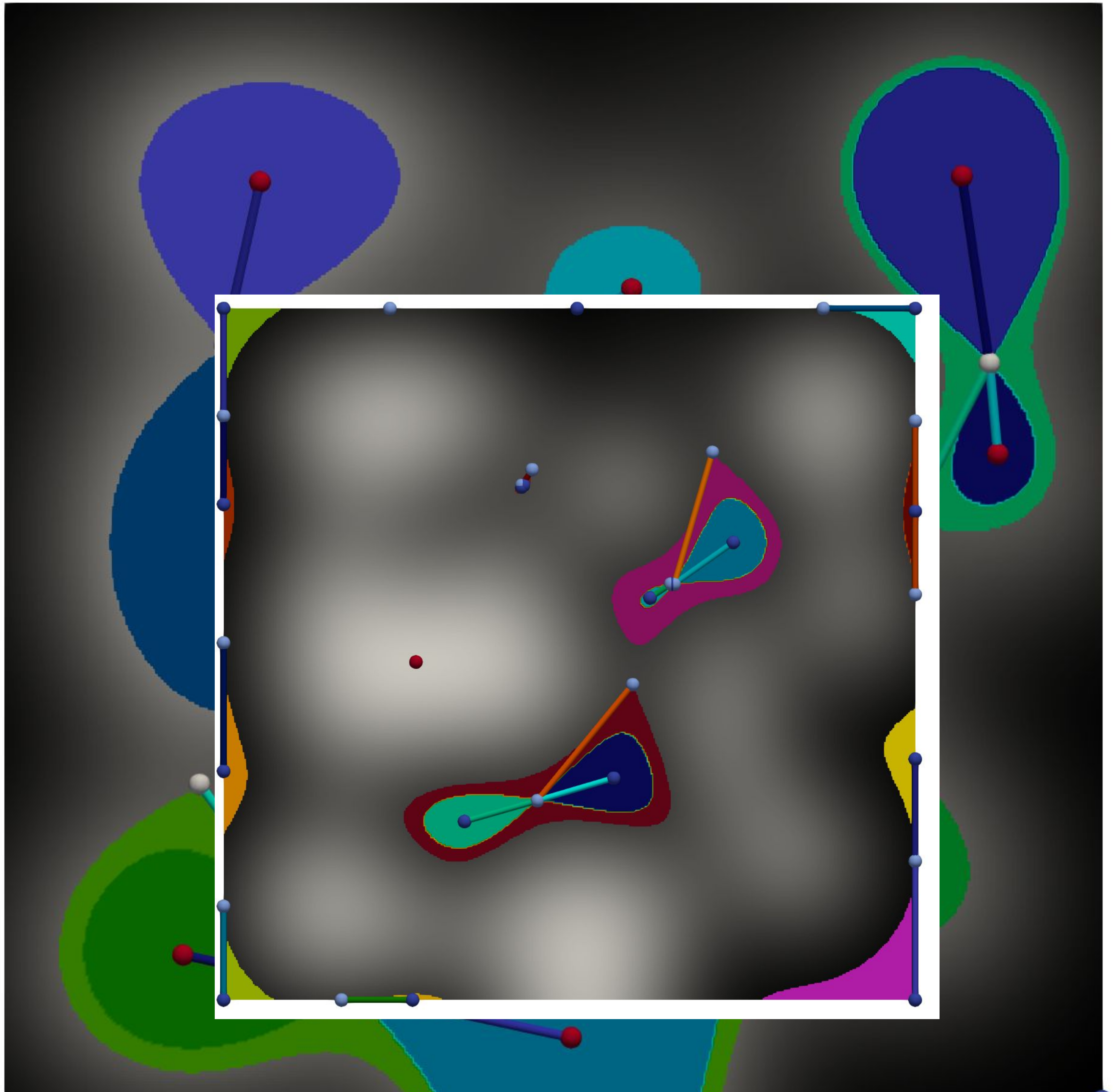
Visualize!!! Visual feedback and debugging is invaluable in converging to a robust feature definition

Topology tells you what *IS*, not necessarily what you think there should be

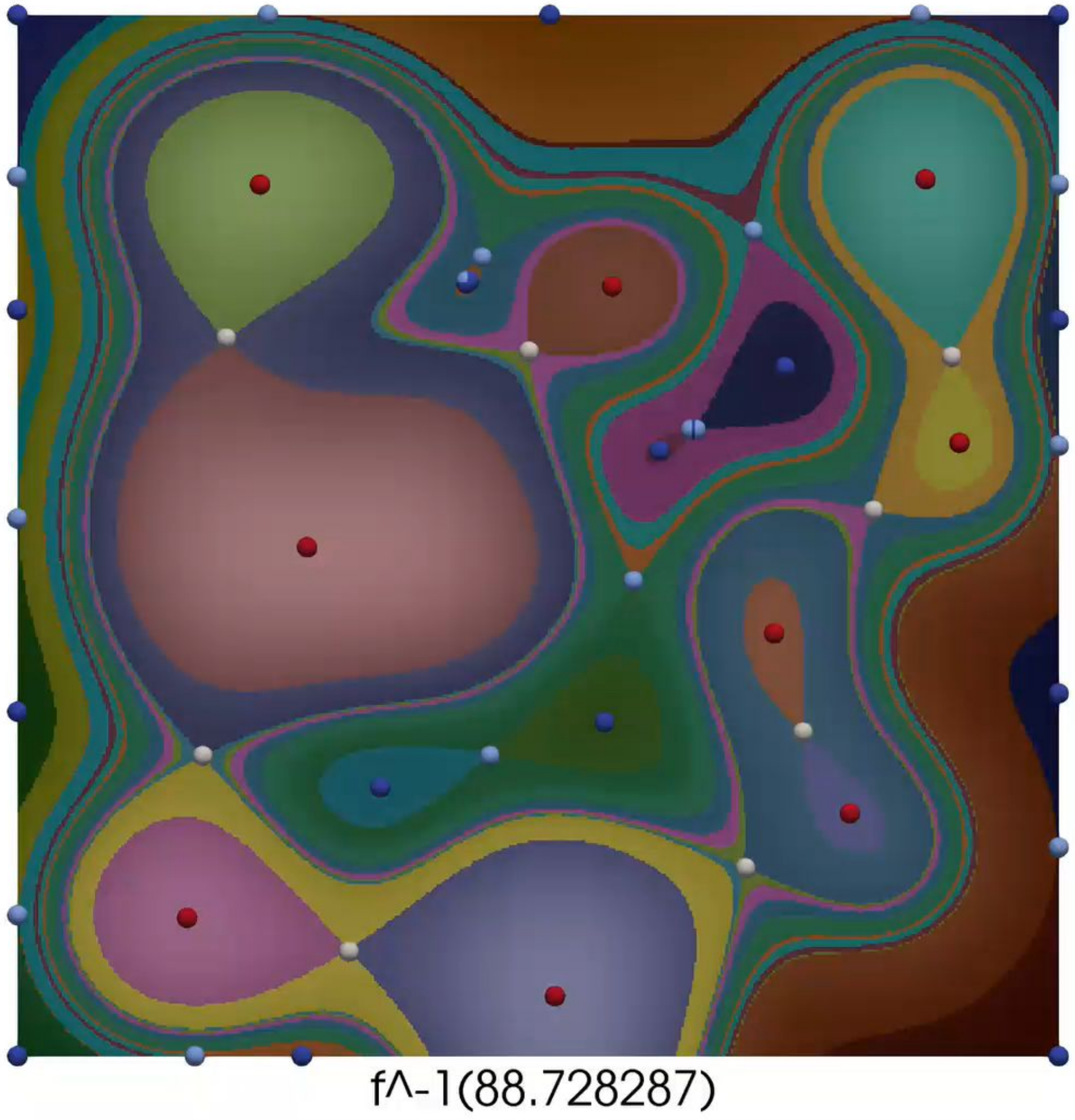


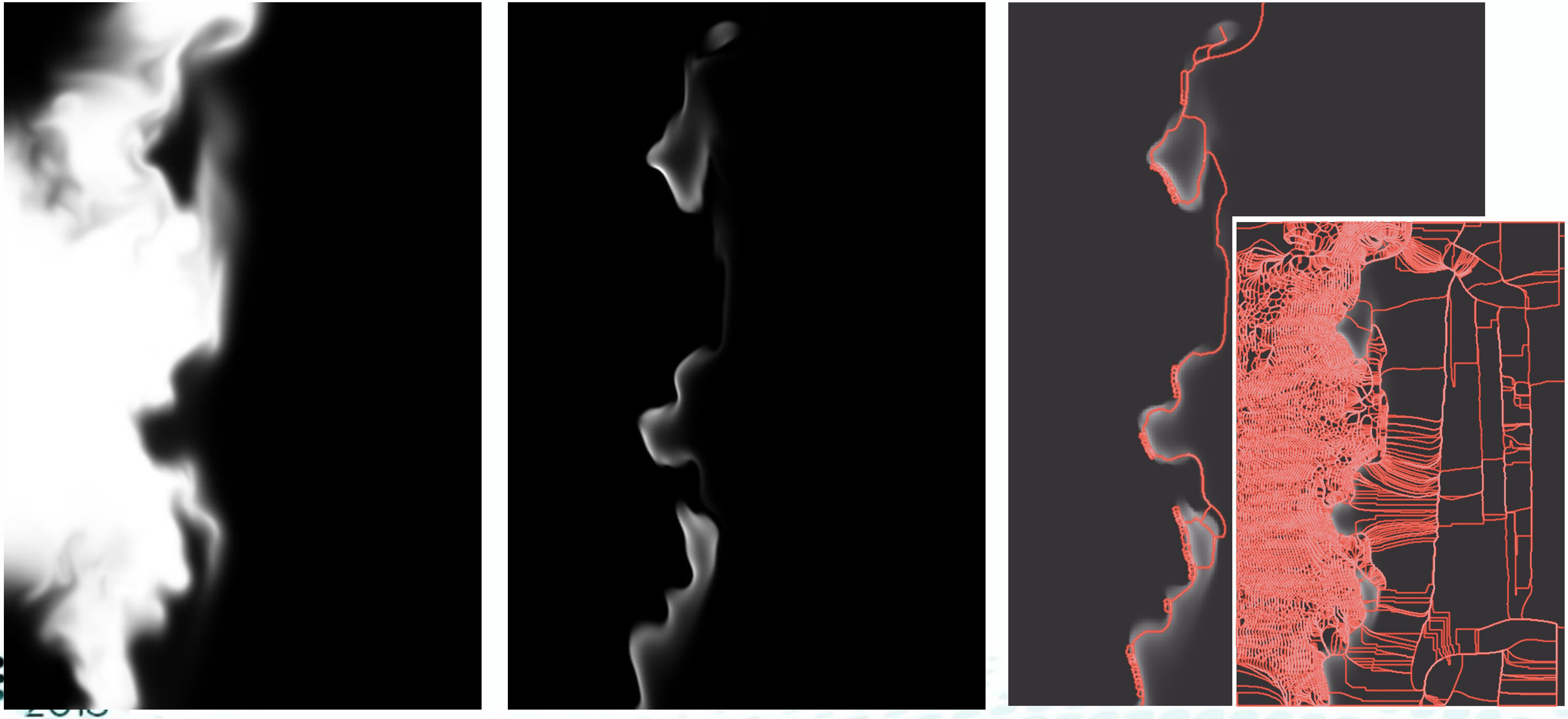
Questions?

jediati@sci.utah.edu



What kinds of abst
available





Crash course on English-> topology

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Translating analysis needs to topology…

What a scientist needs

"I want flame surfaces"

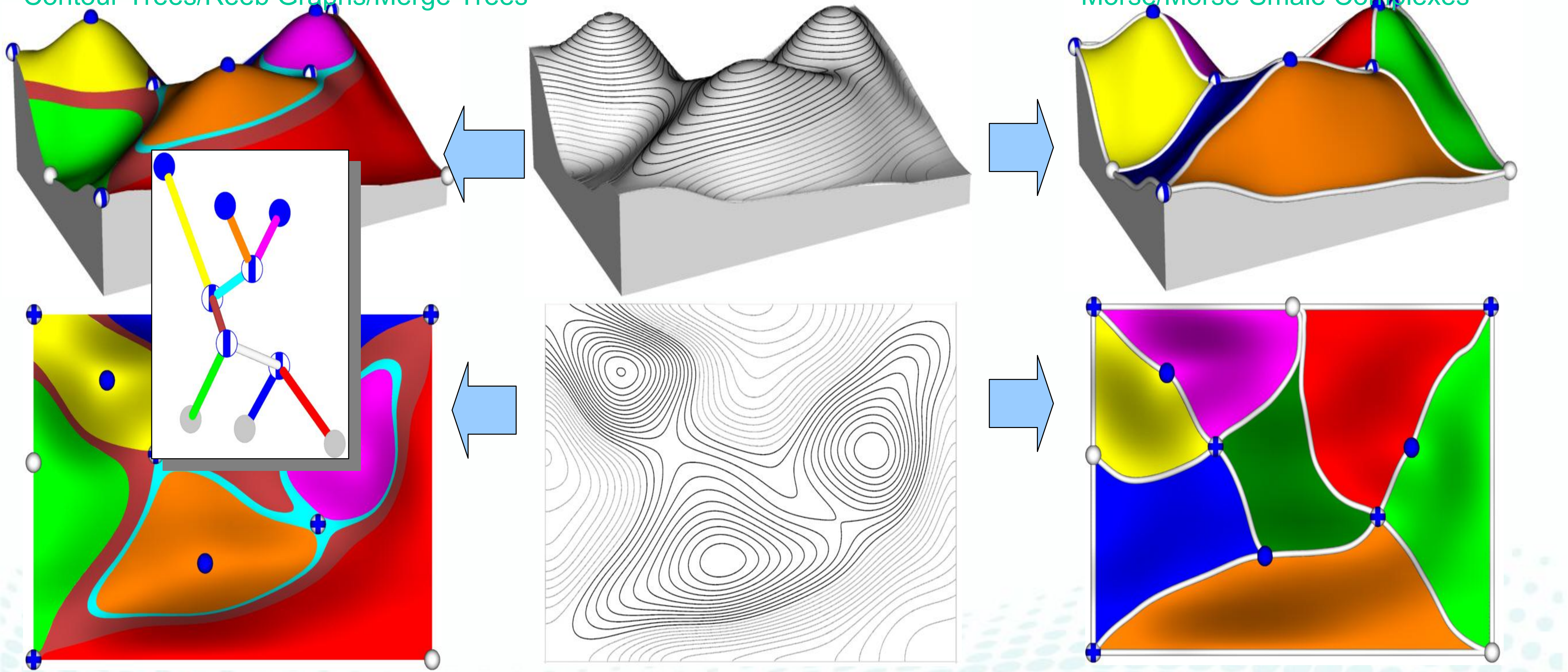
Topological description

High-valued ascending 2-manifolds of temperature field separating fuel and oxidizer

Different abstractions for different kinds of features

Contour Trees/Reeb Graphs/Merge Trees

Morse/Morse-Smale Complexes



Different abstractions for different kinds of features